THE JOURNAL

OF

"The Royal Aeronautical Society"

with which is incorporated "The Institution of Aeronautical Engineers'

Edited for the Council by J. LAURENCE PRITCHARD, Hon. Fellow

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NOTICES

Elections

At a meeting of the Council held on November 15th, 1927, the following were elected:—

Associate Fellows of the Royal Aeronautical Society.—Mr. H. W. Franklin, Mr. M. Shadwell Hooper and Mr. G. Machray.

Associate Members of the Institution of Aeronautical Engineers.—Mr. W. J. Miles, Mr. R. H. Stocken and Mr. A. G. S. Herbert.

Associates.—Mr. T. R. Dowdeswell, Mr. C. B. George, Mr. R. H. S. Mealing and Mr. C. F. Woodgate.

Students.—Mr. L. W. Brewster, Mr. T. T. L. Davies, Mr. L. Ford, Mr. S. Hansel, Mr. S. King and Mr. B. Shenstone.

Member.—Mr. C. H. Pridham.

At the same meeting the Council unanimously elected General A. Guidoni, late Air Attaché at the Italian Embassy, London, an Honorary Fellow.

Chairman

A dinner was given at the Army and Navy Club, on November 8th, by the Council as constituted up to the date of the amalgamation, to Colonel the Master of Sempill, as a mark of appreciation of the immensely valuable services which he has rendered to the Society during the period he has been Chairman, particularly in regard to the negotiations for the amalgamation between the Society and Institution.

Letters of regret for absence were read from Sir Mackenzie Chalmers, Wing Commander T. R. Cave-Browne-Cave, Major D. H. Kennedy, Major A. R. Low and Mr. H. E. Wimperis. Those present at the dinner were:—Colonel the Master of Sempill, Air Vice-Marshal Sir W. Sefton Brancker (President), Captain P. D. Acland, Mr. Griffith Brewer, Mr. A. E. L. Chorlton, Mr. C. R. Fairey, Captain G. T. R. Hill, Mr. J. E. Hodgson, Mr. H. B. Irving, Mr. W. O. Manning, Major R. H. Mayo, Lieut.-Colonel M. O'Gorman, Mr. F. Handley Page, Mr. J. Laurence Pritchard, Major G. H. Scott, Mr. T. O. M. Sopwith, Air Vice-Marshal Sir Vyell Vyvyan, Dr. H. C. Watts and Mr. R. McKinnon Wood.

Index

With this issue of the JOURNAL appears the annual index. This year the index has been entirely re-arranged and very considerably enlarged. It follows the consolidated subject index and will ultimately be incorporated with it.

The attention of members is called to the consolidated subject index. It is a full index of all the matter which has appeared in the JOURNAL from 1897-1926, arranged under subjects and copiously cross-indexed. The index may be obtained from the offices of the Society, price 2s. 6d.

General Guidoni

On the occasion of the departure of General A. Guidoni from London on Monday, November 14th, to take up his new post as Director of Aircraft Construction in Italy, Colonel the Master of Sempill, Chairman, attended on behalf of the Society.

A luncheon was given to General Guidoni by the various air bodies on Friday, 4th November, 1927, at the Savoy Hotel. The Society's representatives were:—Air Vice-Marshal Sir Vyell Vyvyan (Vice-Chairman), Mr. W. L. Bramson, Mr. Griffith Brewer, Colonel M. O'Gorman and Mr. J. Laurence Pritchard.

House Dinner and Informal Discussion

By the courtesy of the Royal Aero Club a most successful dinner and discussion were held at 3, Clifford Street, on Tuesday, November 15th. Sixty-six people attended, a large number of whom took part in the discussion on "Are Aero-Engines Reliable?" A summary of the discussion will be published in a future issue of the JOURNAL.

Students' Section

Mr. W. S. Farren, Associate Fellow, gave the inaugural lecture in the Library on Friday, November 18th, 1927, to the Students' Section, his subject being "Aeroplane Design in the Light of the Lanchester-Prandtl Theory."

A general meeting of the Section was held later at which the following appointments were made:—

Hon. Secretary.—H. F. Winny (East London College).

Committee.—P. G. Thomson (City and Guilds, E. J. Fearn (Napier), J. N. Richmond (Northampton Engineering College), T. Tanner (Imperial College).

Donations

The Council wish to acknowledge gratefully the gift of back numbers of the Society's JOURNAL from Mr. H. D. Carey and Mr. F. P. Walsh, and of a series of volumes of the *R.E. Journal* from Major D. H. Kennedy.

Branches

COVENTRY BRANCH.—A lecture will be given before the Coventry Branch at the Craven Arms Hotel, at 8 p.m., on December 8th, by Mr. A. A. Rubbra, of Messrs. Rolls-Royce, Ltd.

Library

The following books and pamphlets have been received recently and placed in the Library:—

Reports and Memoranda of the Aeronautical Research Committee:—

No. 1081, "The Flexure of Thin Cylindrical Shells and other 'Thin' Sections," by L. G. Brazier.

No. 1097, "The Flow of Air and of an Inviscid Fluid around an Elliptic Cylinder," by A. Fage.

No. 1098, "Distribution of Pressure Over a Monoplane and a Biplane with Wings of Unequal Chord and Equal Span," by A. S. Batson, A. S. Halliday and A. L. Maidens.

No. 1100, "Wind Tunnel Experiments on the Effect on the Maximum Lift of Withdrawing and Discharging Air from the Upper Surface of an Aerofoil," by W. G. A. Perring and G. P. Douglas.

No. 1101, "Full-Scale Tests of a Bristol Fighter with Slot and Aileron Control Operated by a Differential Link Mechanism," by H. M. Garner.

Air Ministry, M.O. 298, Annual Report of the Meteorological Committee to the Air Council for the year ended 31st March, 1927.
Air Publication No. 1278, 1st edition, Jupiter, Series VI.
Professional Paper of the Air Survey Committee, No. 3, "Simple Methods of Surveying from Air Photographs," by M. Hotine.
Official Bulletin No. 12 of the International Commission for Air Navigation.
B.E.S.A. Specification 2 S.P.I. Shackles.
"Aviation," Vol. CXXXI, the Annals of the American Academy of Political and Social Science.
"The Old Flying Days," by C. C. Turner.
Arbeiten zur Luftnavigierung (W.G.L.).
Handbook on Wright Whirlwind Engine.
"Les Bordés Travaillants en Construction Navale et Aeronautique," by L. L. Kahn (Association Technique Maritime et Aeronautique).

Books for Sale

The Society has a number of duplicate books, shown in the following list, on its shelves for sale. Prices may be obtained on application to the Secretary. Early application is necessary as those which remain unsold will be sold *en bloc*:

Aerial Locomotion, by E. H. Harper and Allan Ferguson.
Aerial Russia, The Romance of the Giant Aeroplane, by Lt.-Col. Roustam Bek.
An Airman's Outings, by "Contact."
The Royal Air Force as a Career. Air Ministry Publication.
Aeronautical Meteorology, by Willis Ray Gregg.
A Manual of Rigging for Aircraft. Air Ministry Publication.
How an Aeroplane is Built, by Stepney Blakeney.
Flying and some of its Mysteries, by V. E. Johnson.
Our Atlantic Attempt, by H. G. Hawker and K. Mackenzie Grieve (imperfect).
Aircraft Instruments, by H. N. Eaton, K. Hilding Beij, Wm. G. Brombacher, W. Willard Frymoyer, H. B. Henrickson, C. L. Seward and D. H. Strother.
Edward Teshmaker Busk, A Pioneer in Flight, with a Short Memoir of H. A. Busk, by Mary Busk (two copies).
E. T. Busk, by Mary Busk.
Artificial and Natural Flight, by Sir Hiram Maxim (four copies).
The Problem of Flight, a Text-book of Aerial Engineering, by Herbert Chatley (two copies).
A History of Aeronautics, by E. C. Vivian and Lieut.-Col. W. Lockwood Marsh.
Aircraft Year Book for 1919 and 1921, issued by the Manufacturers' Aircraft Association, U.S.A.
Deutsche Luftfahren-Zeitschrift (bound copy for 1914).
Institut Aerodynamique de Koutchino, Petersbourg, 1905, 1906-09.
Institut Aerodynamique de Koutchino, Petersbourg, Bulletin de Osterrechischf Flug-Zeitschrift, 1911 and 1912.
American National Advisory Committee for Aeronautics Report, 1920, 1915, 1916 (three copies).
Bibliography of Aeronautics, 1909-1910.
Aero Manual, 1912-1913. Repertorie Sportif Technique et Commercial de l'Aeronautique, by Ch. Faroux.
Report of National Advisory Committee for Aeronautics, 1909-1910 (English).
The Air Liner and its Inventor, Alfred W. Lawson, by Cy. Q. Faunce.
Aviatik, by Ing. Wilhelm Kress.
5,000 Miles in a Balloon, by F. Hedges Butler (two copies).

Progress in Flying Machines, by O. Chanute.
Aeronautics, An Abridgment of Aeronautical Specifications filed at the Patent Office from A.D. 1815 to A.D. 1891, by Griffith Brewer and Patrick Y. Alexander.
Flying, Some Practical Experiences, by Gustav Hamel and C. C. Turner.
The Dominion of the Air, by the Rev. J. M. Bacon.
The Conquest of the Air, by John Alexander, with preface by Sir Hiram Maxim.
Model Balloons and Flying Machines, by J. H. Alexander.
Abridgments of Specifications (Aeronautics), 1877-83.
Travels in the Air, by James Glaisher, Camille Flammarion, W. de Fonvielle and Gaston Tissandier.
Notre Flotte Aerienne, par W. de Fonvielle et Georges Besancon.
Luftschrauben Leiftaden fur den Bau und die Behandlung von Propellern, by Von Paul Bejeuhr.
Wie Berechnet, Konstruiert und baut man ein Flugzeug, Von Dipl. Ing. O. L. Skopik.
Die Luftschiffahrt, by Moedebeck, 1900.
Airscrews in Theory and Experiment, by A. Fage.
Aviation Engines, by V. W. Page.
Air Power, Naval, Military, Commercial, by C. Graham White and Harry Harper.
Airscrews, by M. A. S. Riach.
The Complete Airman, by G. C. Bailey.
Elementary Aeronautics, by A. P. Thurston.
History and Practice of Aerostation, by Tiberius Cavallo.
Aeronautica or Sketches Illustrative of Theory and Practice of Aerostation, by Monck Mason (two copies).
Flying Machines, Past, Present and Future, by A. W. Marshall and Henry Greenly.
Aircraft in Warfare, by F. W. Lanchester.
Leight Flugzeugbau, by Dr. Ing. G. Lachmann.
The Aeronautical Annual, 1896 (two copies), 1895, 1897.
Aircraft in Peace and the Law, by J. M. Spaight.
Aeroplane Design, by F. S. Barnwell and a Simple Explanation of Inherent Stability, by W. H. Sayers (one book).
Astra Castra, Experiments and Adventures in the Atmosphere, by Hatton Turnor (three copies).
Experiments in Aerodynamics, by S. P. Langley.
The Internal Work of the Wind, by S. P. Langley.
Wonderful Balloon Ascents or the Conquest of the Skies, from the French of F. Marion.
The Flying Machine, Two Papers, "The Aerofoil" and "The Screw Propeller," by F. W. Lanchester.
The Dynamics of the Aeroplane, by Rene Devillers, translated by J. Walker.
Les Cerfs-Volants, par J. Lecornu.
Bulletin de la Federation Aeronautique Internationale, No. 27-28, Dec., 1926.
Eleventh Annual Report of the National Advisory Committee for Aeronautics (American), 1925.
Tenth Annual Report of the National Advisory Committee for Aeronautics (American), 1924.
Twelfth Annual Report of the National Advisory Committee for Aeronautics (American), 1926. (Administrative only.)
American National Advisory Committee for Aeronautics, No. 123, 1921.
American National Advisory Committee for Aeronautics, No. 139, 1922.

Advisory Committee for Aeronautics. Reports and Memoranda No. 19, 1910.
La Resistance de l'Air et Aviation. Eiffel. 1911.
La Resistance de l'Air des Formules et des Experiences. Eiffel. 1910.
Advisory Committee for Aeronautics. 1909-1914. (Annual Reports.)
Ditto, No. 196, June, 1915.
Ditto, No. 272, May, 1916.
Ditto, No. 283, Aug., 1916.
Ditto, No. 323, Dec., 1916.
Ditto, No. 353, April, 1917.
Ditto, No. 355, Oct., 1917.
Ditto, No. 366, Dec., 1917.
Ditto, No. 375, Jan., 1918.
Ditto, No. 377, Oct., 1917.
Ditto, No. 388, Dec., 1917.
Ditto, No. 394, Jan., 1918.
Ditto, No. 415, Jan., 1918.
Ditto, No. 432, April, 1918.
Ditto, No. 440, March, 1918.
Ditto, No. 449, May, 1918.
Ditto, No. 469, June, 1918.
Ditto, No. 481, Sept., 1918.
Ditto, No. 531, March, 1918.
Ditto, No. 545, Oct., 1918.
Ditto, No. 546, Oct., 1918.

Forthcoming Arrangements

Thursday, December 1st, 6.30 p.m.—Major J. D. Rennie, A.R.T.C., A.M.Inst.C.E., F.R.Ae.S., "The Problem of the Long-Range Flying Boat."

Thursday, December 8th, 6.30 p.m.—Captain F. Entwistle, B.Sc., "Fog."

Thursday, December 15th, 6.30 p.m.—Major F. A. Bumpus, "The Development of Torpedo-Carrying Aircraft."

Thursday, January 5th, 1928, 6.30 p.m.—Mr. A. Fage, A.R.C.Sc., F.R.Ae.S., "Some Recent Experiments on Fluid Motion."

All the lectures will take place at the Royal Society of Arts, Adelphi, W.C.2.

Lecture Programme—Second Half of Session, 1928

The following is the second half of the programme of lectures for the present Session. All the lectures will be held in the Theatre of the Royal Society of Arts, 18, John Street, Adelphi, W.C.2, at 6.30 p.m., except the Joint Meeting with the Institution of Automobile Engineers, which will take place at 7.45 p.m.:

Jan. 5.—Mr. A. Fage, A.R.C.Sc., F.R.Ae.S., "Some Recent Experiments on Fuid Motion."

Jan. 19.—Major W. S. Tucker, "The Problem of Noise in Civil Aircraft and Possibilities of its Elimination."

Jan. 26.—Mr. R. J. Mitchell, A.F.R.Ae.S., Mr. P. A. Ralli, A.F.R.Ae.S., Captain G. S. Wilkinson, A.F.R.Ae.S., "Schneider Trophy Machine Design."

Feb. 2.—Major H. N. Wylie, F.R.Ae.S., "The Design and Production of Steel Aircraft."

Feb. 7 (Tuesday).—Wing Cmdr. I. G. V. Fowler, A.F.C., "The Maintenance and Repair of Aero Engines." Joint Meeting with I.A.E.

Feb. 16.—Mr. G. P. Douglas, M.C., D.Sc., "Experiments on Model Airscrews at High Tip Speeds."

Mar. 1.—Wing Cmdr. R. M. Hill, M.C., A.F.C., F.R.Ae.S., "Experiences with the Baghdad Air Mail."
Mar. 15.—Dr. Rudorf, "Testing of Materials Used in Aircraft Construction."
April 12.—Mr. G. R. Volkert, A.F.R.Ae.S., "Some Aspects of the Development of the Slot."
April 26.—Mr. B. N. Wallis, "The Design and Construction of Modern Rigid Airships."

R.38 Memorial Prize

The closing date for the receipt of names of entrants for the R.38 Memorial Prize, 1928, is December 31st, 1927. The prize is offered annually for the best paper received by the Society on some subject of a technical nature in the science of aeronautics. Other things being equal, preference will be given to papers which relate to Airships. The prize is open to international competition and is of the value of twenty-five guineas.

SECONDARY STRESSES IN AIRSHIP HULL STRUCTURES

BY J. F. BAKER, M.A.

Introduction

In no branch of structural engineering is there a rigorous definition of the term "secondary stress."

If the primary stresses in a structure are determined, that is, the stresses in the members due to an external load system, assuming that all the members are joined together by perfect pin- or ball-joints, then the secondary stresses are in general taken to be the additional stresses due to the rigidity of the actual joints used in practice.

In such a highly redundant structure as an airship hull the labour involved, in determining even the primary stresses, precludes the use of the normal methods of stressing. It is usual to make use of generalised methods which give approximate results. These generalised methods imply that the external loads are applied to the structure in a certain distribution. Though this is rarely achieved, yet the results obtained are in most cases sufficiently accurate if suitable bracing is supplied to redistribute the external loads over the cross-section; the effect of the initial wrong distribution being then merely local.

These local effects may be included as secondary stresses, and it is proposed in the first part of this paper to examine a possible distribution of external load which may cause large secondary stresses of this nature.

The more orthodox problem of the effect of rigid joints is present in airship hulls, the continuity of the longitudinals over the transverse frames providing rigid joints, which may cause serious redistribution of load in the members. In this case there is no inherent difficulty in the problem, but satisfactory approximate methods must be found to cut down the enormous amount of labour due to the complexity of the structure. In Part II. a method is described and a case worked through from which an estimate of the labour involved can be made.

Wherever possible, the methods and nomenclature of the "Report of the Airship Stressing Panel" (R. & M. 800) have been used in this paper.

PART I.

Secondary Stresses Due to the Distribution of External Longitudinal Loads

The methods employed in determining the primary stresses in an airship hull depend upon a knowledge of the resultant action at each cross-section. For the results given by these methods to be exact the external loads, causing this resultant action, must be imposed in some specified manner; this is not, in practice, always the case, but in general the type of construction is such that the discrepancy between the actual distribution of external load and that demanded by theory gives rise merely to small local differences.

An airship hull consists of a number of polygonal transverse frames formed into a tubular structure by longitudinal girders; the panels between consecutive frames and longitudinals are braced. For the consideration of primary stresses, all external loads are assumed to act at the joints of the structure, that is, where the longitudinals cross the transverse frames. The external loads may, therefore, when the ship is on an even keel, be taken to act at, and in the plane of, the

transverse frame. The frame is so built (in the case of the Zeppelin type, by very complete bracing in the plane of the frame) that these loads are distributed, approximately, in the way that the tubular structure demands, and so the loads resulting are not seriously different from those given by theory.

Due to certain causes, however, external loads may be applied at right angles to the plane of the frame, and if these are not distributed in the specified manner difficulties may arise, as the frame in this position is not an efficient means of bringing about that correct specified distribution.

If a system of forces acts perpendicular to the plane of the frame, it may be replaced by a system giving the same resultant actions as the original and obeying the distribution demanded by theory, with the addition of a system of forces giving rise to no resultant actions. It is the effect of the latter system, with no resultant actions, which must be examined.

The stiff polygonal nose-caps, now fitted to airships for mast mooring, may give rise to such a system of forces, as that under consideration, when a transverse force is applied to the nose.

It would be impossible to stress an actual airship structure by the exact methods demanded by the problem, owing to the amount of work involved, but there is nothing to be gained by complicating the structure considered. It is only necessary that it should correspond to an airship hull in its main features; that is, it should be a braced tubular structure with a keel member differing in cross-section from the other longitudinals.

The external loads must be applied at the joints; the simplest symmetrical form, therefore, would be a tube with a quadrilateral cross-section; this has not been taken as it does not allow that freedom of distribution of external load which is desirable.

The particular structure chosen is a hexagonal braced tube, shown in Figs. 1 (Pt. I.) and 2 (Pt. I.). The longitudinal members are all similar with the exception of the keel ($A_1B_1C_1D_1$, etc.) which has a different area. The diagonal cross-bracing members are all similar; the free bulkhead "A" is radially braced with members which, with all transverse members, are assumed to be inextensible. There is no radial bracing in the transverse frames, with the exception of the first. The structure is N bays long and is attached to a wall at the end of the Nth bay; this cross-section is, therefore, kept plane.

The structure is redundant, with $(6N+4)$ redundancies, and the usual methods of strain energy are used to find the loads in the members under a given external load system.

The method of presentation follows exactly that used in R. & M. (800), App. (5), in which a similar structure is analysed for a shear applied at the free end.

The external load system consists of forces P_1, P_2, P_3, . . . P_6, acting at the joints A_1, A_2, A_3, . . . A_6, in the directions of the longitudinals as shown in Fig. 1 (Pt. I.).

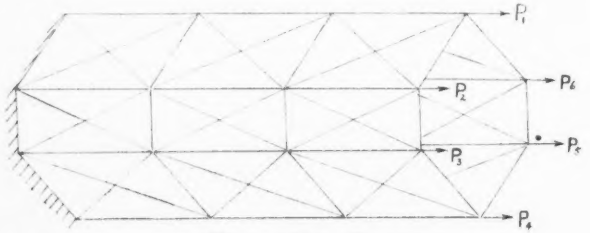


FIG. 1, PART I.

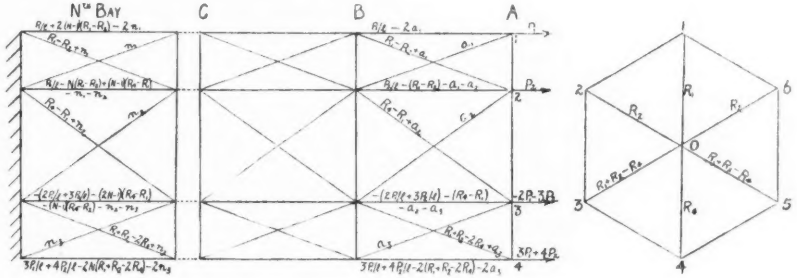
This system is to be symmetrical about the axis A_1A_4, to have no resultant moment and no resultant axial force.

These conditions give the relations

$$P_2 = P_6 \text{ and } P_3 = P_5$$
$$P_1 + 2P_2 + 2P_3 + P_4 = 0$$
$$4P_1 + 6P_2 + 2P_3 = 0$$

which give

$$P_3 = -2P_1 - 3P_2$$
$$P_4 = 3P_1 + 4P_2$$



TENSION COEFFICIENTS IN N^TH BAY.

LONGS.	11	$R/\ell + 2(N-1)(R-R_1) - 2n_1$
	22	$R/\ell - N(R-R_1) + (N-1)(R_4-R) - n_2 - n_4$
	33	$-(2P_2+3P_3)/\ell - (2N-1)(R_4-R) - (N-1)(R_4-R_2) - n_4 - n_3$
	44	$(3P_2+4P_3)/\ell - 2N(R_1+R_2-2R_4) - 2n_3$
DIAGS.	12	n_1
	21	$R_1 - R_2 + n_1$
	23	n_4
	32	$R_4 - R_2 + n_4$
	34	n_3
	43	$R_1 + R_2 - 2R_4 + n_3$

FIG. 2, PART I.

Fig. 2 gives the tension coefficients in the longitudinals, the diagonal bracing members and the radial members.

The tension coefficient of a member is the load in that member divided by its length, and the sign convention adopted is plus for tension and minus for compression.

The tension coefficients of the redundant diagonal bracing members are taken to be a_1, a_2, a_3, b_1, b_2, b_3, . . . n_1, n_2, n_3, in the first, second, up to the nth bay as shown. The redundant radial members are denoted by R_1, R_2 and R_4. Taking the symmetry of the system into account, the tension coefficient in any other member may then be written down in terms of the external loads and the redundancies $(R_1 - R_4)$, $(R_2 - R_4)$, a_1, a_2, a_3, b_1, b_2, b_3, . . . n_1, n_2, n_3. These will therefore yield $3(N+2)$ equations for their evaluation.*

Suffix d will denote diagonal members.

L will denote keel members.

l will denote any other longitudinal.

The quantity $S_d E_d / l^3_d$ will be written Ω_d, where S is the cross-sectional area, E the Young's modulus and l the length of the member. In the expressions P_1/l, P_2/l, l is the length of a longitudinal.

* Equations (1) and (2) are obtained by equating to zero, the differential coefficients of the total internal work, with regard to R_1 and R_2 respectively.

The $(3N+2)$ strain energy equations are then—

$$(R_1-R_4)[\{N(N+1)(2N+1)/3\}(1/\Omega_L-1/\Omega_1)+4N^3/\Omega_1+3N/\Omega_d]$$
$$+(R_2-R_4)[\{N(N+1)(2N+1)/3\}(1/\Omega_L-1\Omega_1)+(N^2/\Omega_1]$$
$$+(a_1+b_1+c_1+\ldots+n_1)[1/\Omega_1+1/\Omega_d]-(a_2+b_2+c_2+\ldots+n_2)[1/\Omega_d]$$
$$+a_3[2(1/\Omega_L-1/\Omega_1)+1/\Omega_1+1/\Omega_d]$$
$$+b_3[4(1/\Omega_L-1/\Omega_1)+1/\Omega_1+1/\Omega_d]+\ldots+n^3[2N(1/\Omega_L-1/\Omega_1)+1/\Omega_1+1/\Omega_d]$$
$$-(P_1/2l)[N(3N+1)/\Omega_1+3N(N+1)/\Omega_L]-(P_2/2l)[8N^2/\Omega_1+4N(N+1)/\Omega_L]=0 \quad (1)$$

$$(R_1-R_4)[\{N(N+1)(2N+1)/3\}(1/\Omega_L-1/\Omega_1)+N^2/\Omega_1]$$
$$+(R_2-R_4)[\{N(N+1)(2N+1)/3\}(1/\Omega_L-1/\Omega_1)+N(2N^2+1)/\Omega_1+2N/\Omega_d]$$
$$+a_1[-1/\Omega_1-1/\Omega_d]+b_1[-1/\Omega_d]+c_1[1/\Omega_1-1/\Omega_d]+\ldots+n_1[(N-2)/\Omega_1-1/\Omega_d]$$
$$-a_2[1/\Omega_1]-b_2[3/\Omega_1]-\ldots-n_2[(2N-1)/\Omega_1]$$
$$+a_3[2(1/\Omega_L-1/\Omega_1)+2/\Omega_1+1/\Omega_d]$$
$$+b_3[4(1/\Omega_L-1/\Omega_1)+3/\Omega_1+1/\Omega_d]+\ldots$$
$$+n_3[2N(1/\Omega_L-1/\Omega_1)+(N+1)/\Omega_1+1/\Omega_d]$$
$$-(P_1/2l)[3N(N-1)/\Omega_1+3N(N+1)/\Omega_L]$$
$$-(P_2/2l)[2N(N-2)/\Omega_1+4N(N+1)/\Omega_L]=0 \quad \ldots \quad (2)$$

$$(R_1-R_4)[1/\Omega_1+1\Omega_d]+(R_2-R_4)[-1/\Omega_1-1/\Omega_d]-(P_1/l)[1/\Omega_1]-(P_2/l)[1/\Omega_1]$$
$$+a_1[3/\Omega_1+2/\Omega_d]+a_2[1/\Omega_1]=0 \quad \ldots \quad (3)$$

$$(R_1-R_4)[1/\Omega_d]+(R_2-R_4)[1/\Omega_1]+(P_2/l)[1/\Omega_1]-\{(2P_1+3P_2)/l\}[1/\Omega_1]$$
$$-a_1[1/\Omega_1]-2a_2[1/\Omega_1+1/\Omega_d]-a_3[1/\Omega_1]=0 \quad \ldots \quad (4)$$

$$(R_1-R_4)[2(1/\Omega_L-1/\Omega_1)+1/\Omega_1+1/\Omega_d]$$
$$+(R_2-R_4)[2(1/\Omega_L-1/\Omega_1)+2/\Omega_1+1/\Omega_d]+\{(2P_1+3P_2)/l\}[1/\Omega_1]$$
$$-\{(3P_1+4P_2)/l\}[1/\Omega_L]+a_2[1/\Omega_1]+a_2[2(1/\Omega_L-1/\Omega_1)+3/\Omega_1+2/\Omega_d]=0 \quad (5)$$

$$(R_1-R_4)[1/\Omega_1+1/\Omega_d]+(R_2-R_4)[-1/\Omega_d]-(P_1/l)[1/\Omega_1]-(P_2/l)[1/\Omega_1]$$
$$+b_1[3/\Omega_1+2/\Omega_d]+b_2[1/\Omega_1]=0 \quad \ldots \quad (6)$$

$$(R_1-R_4)[1/\Omega_d]+(R_2-R_4)[3/\Omega_1]+(P_2/l)[1/\Omega_1]-\{(2P_1+3P_2)/l\}[1/\Omega_1]$$
$$-b_1[1/\Omega_1]-2b_2[1/\Omega_1+1/\Omega_d]-b_3[1/\Omega_1]=0 \quad \ldots \quad (7)$$

$$(R_1-R_4)[4(1/\Omega_L-1/\Omega_1)+1/\Omega_1+1/\Omega_d]$$
$$+(R_2-R_4)[4(1/\Omega_L-1/\Omega_1)+3/\Omega_1+1/\Omega_d]$$
$$+\{(2P_1+3P_2)/l\}[1/\Omega_1]-\{(3P_1+4P_2)/l\}[1/\Omega_L]+b_2[1/\Omega_1]$$
$$+b_3[2(1/\Omega_L-1/\Omega_1)+3/\Omega_1+2/\Omega_d]=0 \quad \ldots \quad (8)$$

. . . . .

$$(R_1-R_4)[1/\Omega_1+1/\Omega_d]+(R_2-R_4)[(N-2)/\Omega_1-1/\Omega_d]-(P_1/l)[1/\Omega_1]-(P_2/l)[1/\Omega_1]$$
$$+n_1[3/\Omega_1+2/\Omega_d]+n_2[1/\Omega_1]=0 \quad \ldots \quad (3N)$$

$$(R_1-R_4)[1/\Omega_d]+(R_2-R_4)[(2N-1)/\Omega_1]+(P_2/l)[1/\Omega_1]-\{(2P_1+3P_2)/l\}[1/\Omega_1]$$
$$-n_1[1/\Omega_1]-2n_2[1/\Omega_1+1/\Omega_d]-n_3[1/\Omega_1]=0 \quad \ldots \quad (3N+1)$$

$$(R_1-R_4)[2N(1/\Omega_L-1/\Omega_1)+1/\Omega_1+1/\Omega_d]$$
$$+(R_2-R_4)[2N(1/\Omega_L-1/\Omega_1)+(N+1)/\Omega_1+1/\Omega_d]$$
$$+\{(2P_1+3P_2)/l\}[1/\Omega_1]-\{(3P_1+4P_2)/l\}[1/\Omega_L]$$
$$+n_2[1/\Omega_1]+n_3[2(1/\Omega_L-1/\Omega_1)+3/\Omega_1+2/\Omega_d]=0 \quad \ldots \quad (3N+2)$$

If the corresponding equations (3), (6), . . . (3N); (4), (7), . . . $(3N+1)$; (5), (8), . . . $(3N+2)$; are compared, it will be seen that a relation exists between a_1, b_1, . . . n_1; a_2, b_2, . . . n_2; a_3, b_3, . . . n_3; which may be expressed as

$$n_1=a_1+2(N-1)a_1$$
$$n_2=a_2+2(N-1)a_2$$
$$n_3=a_3+2(N-1)a_3$$

giving $b_1=a_1+2a_1$, $c_1=a_1+4a_1$, etc.

Making use of these relations, equations (1) and (2) may be reduced to the form of equations (4a) and (5a) below and the last (3N) equations above, to the following three :—

$$(R_2-R_4)\,[1/\Omega_1]+2a_1\,[3/\Omega_1+2/\Omega_d]+2a_2\,[1/\Omega_1]=0 \quad . \quad . \quad (1a)$$

$$(R_2-R_4)\,[2/\Omega_1]-2a_1\,[1/\Omega_1]-4a_2\,[1/\Omega_1+1/\Omega_d]-2a_3\,[1/\Omega_1]=0 \quad . \quad (2a)$$

$$(R_1-R_4)\,[2\,(1/\Omega_L-1/\Omega_1)]+(R_2-R_4)\,[2\,(1/\Omega_L-1/\Omega_1)+1/\Omega_1] + 2a_2\,[1/\Omega_1]+2a_3\,[2\,(1/\Omega_L-1/\Omega_1)+3/\Omega_1+2\Omega_d]=0 \quad . \quad . \quad (3a)$$

From which

$$a_1=-(1/A\Omega_1)\,[(R_1-R_4)\,(1/\Omega_L-1/\Omega_1) + (R_2-R_4)\,(1/\Omega_1+5/\Omega_d+5/\Omega_L+2\Omega_1/\Omega_L\Omega_d+2\Omega_1/\Omega_d^2)]$$

$$a_2=-(1/A\Omega_1)\,[(R_1-R_4)\,(3/\Omega_1+2/\Omega_d-3/\Omega_L-2\Omega_1/\Omega_L\Omega_d) + (R_2-R_4)\,(-2/\Omega_1-8/\Omega_d-10/\Omega_L-6\Omega_1/\Omega_L\Omega_d-4\Omega_1/\Omega_d^2)]$$

$$a_3=-(1/A\Omega_1)\,[(R_1-R_4)\,(-5/\Omega_1-10/\Omega_d+5/\Omega_L-4\Omega_1/\Omega_d^2+10\Omega_1/\Omega_L\Omega_d+4\Omega_1^2/\Omega_d^2\Omega_L) + (R_2-R_4)\,(1/\Omega_1-3/\Omega_d+5/\Omega_L-2\Omega_1/\Omega_d^2+10\Omega_1/\Omega_L\Omega_d+4\Omega_1^2/\Omega_d^2\Omega_L)]$$

Where

$$A=(2/\Omega_1)\,[1/\Omega_1+9/\Omega_d+5/\Omega_L+10\Omega_1/\Omega_d\Omega_L+12\Omega_1/\Omega_d^2+4\Omega_1^2/\Omega_d^2\Omega_L+4\Omega_1^2/\Omega_d^3]$$

By subtracting equations (1a), (2a), (3a) from the corresponding equations (3), (4), (5), it is possible to obtain three equations in (a_1-a_1), (a_2-a_2), (a_3-a_3), which give

$$(a_1-a_1)=(\tfrac{1}{2})\,\{-(R_1-R_4)+(R_2-R_4)\} + (1/A\Omega_1)\,[(P_1/l)\,(1/\Omega_1+10/\Omega_d+11/\Omega_L+4\Omega_1/\Omega_d\Omega_L+4\Omega_1/\Omega_d^2) + (P_2/l)\,(10/\Omega_d+12/\Omega_L+4\Omega_1/\Omega_d\Omega_L+4\Omega_1/\Omega_d^2)]$$

$$(a_2-a_2)=(\tfrac{1}{2})\,(R_1-R_4)-(1/A\Omega_1)\,[(P_1/l)\,(1/\Omega_1+14/\Omega_d+23/\Omega_L+14\Omega_1/\Omega_d\Omega_L+8\Omega_1/\Omega_d^2) + (P_2/l)\,(-2/\Omega_1+12/\Omega_d+26/\Omega_L+16\Omega_1/\Omega_d\Omega_L+8\Omega_1/\Omega_d^2)]$$

$$(a_3-a_3)=-(\tfrac{1}{2})\,\{(R_1-R_4)+(R_2-R_4)\}+(1/A\Omega_1)\,[(P_1/l)\,(-3/\Omega_1 - 16/\Omega_d+15/\Omega_L+30\Omega_1/\Omega_d\Omega_L-8\Omega_1/\Omega_d^2+12\Omega_1^2/\Omega_d^2\Omega_L) + (P_2/l)\,(-8/\Omega_1-26/\Omega_d+20/\Omega_L+40\Omega_1/\Omega_d\Omega_L-12\Omega_1/\Omega_d^2+16\Omega_1^2/\Omega_d^2\Omega_L)]$$

From these values of a and a equations (4a) and (5a), which were reduced from equations (1) and (2), may be solved for (R_1-R_4) and (R_2-R_4).

$$(R_1-R_4)\,[\{N(N+1)(2N+1)/3\}(1/\Omega_L-1/\Omega_1)+4N^3/\Omega_1+3N/\Omega_d] + (R_2-R_4)\,[\{N(N+1)(2N+1)/3\}(1/\Omega_L-1/\Omega_1)+N^2/\Omega_1] + a_1\,[N(1/\Omega_1+1/\Omega_d)]-a_2\,[N/\Omega_d]+a_3\,[N(N+1)(1/\Omega_L-1/\Omega_1)+N/\Omega_1+N/\Omega_d] + a_1\,[N(N-1)(1/\Omega_1+1/\Omega_d)]-a_2\,[N(N-1)(1/\Omega_d)] + a_3\,[4/3\,N(N^2-1)(1/\Omega_L-1/\Omega_1)+N(N-1)(1/\Omega_1+\Omega_d)] - (P_1/2l)\,[N(3N+1)/\Omega_1+3N(N+1)/\Omega_L]-(P_2/2l)\,[8N^2/\Omega_1+4N(N+1)/\Omega_L]=0 \quad (4a)$$

$$(R_1-R_4)\,[\{N(N+1)(2N+1)/3\}(1/\Omega_L-1/\Omega_1)+N^2/\Omega_1] + (R_2-R_4)\,[\{N(N+1)(2N+1)/3\}(1/\Omega_L-1/\Omega_1)+N(2N^2+1)/\Omega_1+2N/\Omega_d] + a_1\,[(N^2-3N)/2\Omega_1-N/\Omega_d]-a_2\,(N^2/\Omega_1) + a_3\,[N(N+1)(1/\Omega_L-1/\Omega_1)+N(N+3)/2\Omega_1+N/\Omega_d] + a_1\,[2N(N^2-3N+2)/3\Omega_1-(N^2-N)/\Omega_d]-a_2\,[N(4N^2-3N-1)/3\Omega_1] + a_3\,[\{4N(N^2-1)/3\}(1/\Omega_L-1/\Omega_1)+N(2N^2+3N-5)/3\Omega_1+N(N-1)/\Omega_d] - (P_1/2l)\,[3N(N-1)/\Omega_1+3N(N+1)/\Omega_L] - (P_2/2l)\,[2N(N-2)/\Omega_1+4N(N+1)/\Omega_L]=0 \quad . \quad . \quad (5a)$$

Giving

$$(R_1-R_4)=[-6/\{N^4(r_1r_2-16t^2)+N^2(r_1s_2+r_2s_1+8t^2)+(s_1s_2-t^2)\}] [(P_1/l)\{N^3(p_1r_2-4tp_2)+N(p_1s_2+tp_2)\} + (P_2/l)\{N^3(q_1r_2-4tq_2)+N(q_1s_2+tq_2)\}]$$

$$(R_2-R_4)=[-6/\{N^4(r_1r_2-16t^2)+N^2(r_2s_2+r_2s_1+8t^2)+(s_1s_2-t^2)\}] [(P_1/l)\{N^3(r_1p_2-4tp_1)+N(s_1p_2+tp_1)\} + (P_2/l)\{N^3(r_1q_2-4tq_1)+N(s_1q_2+tq_1)\}]$$

where

$$p_1=(1/\Omega_l)\,[-11/\Omega_d-36/\Omega_L-103\Omega_l/\Omega_d\Omega_L-28/\Omega_l/\Omega_d{}^2 -68\Omega_l{}^2/\Omega_d{}^2\Omega_L-12\Omega_l{}^2/\Omega_d{}^3-12\Omega_l{}^3/\Omega_d{}^3\Omega_L]$$

$$q_1=(1/\Omega_l)\,[-46/\Omega_d-72/\Omega_L-182\Omega_l/\Omega_d\Omega_L-84\Omega_l/\Omega_d{}^2 -108\Omega_l{}^2/\Omega_d{}^2\Omega_L-32\Omega_l{}^2/\Omega_d{}^3-16\Omega_l{}^3/\Omega_d{}^3\Omega_L]$$

$$r_1=[160\Omega_l/\Omega_d{}^3+632/\Omega_d\Omega_L+288/\Omega_l\Omega_L+320\Omega_l/\Omega_d{}^2\Omega_L +32\Omega_l{}^2/\Omega_d{}^3\Omega_L+280/\Omega_l\Omega_d+448/\Omega_d{}^2]$$

$$s_1=[176\Omega_l/\Omega_d{}^3-68/\Omega_d\Omega_L-72/\Omega_l\Omega_L+100\Omega_l/\Omega_d{}^2\Omega_L +64\Omega_l{}^2/\Omega_d{}^3\Omega_L+52/\Omega_l\Omega_d+50/\Omega_d{}^2+72\Omega_l{}^2/\Omega_d{}^4]$$

$$t=(1/\Omega_d)\,[-24/\Omega_l-28/\Omega_d+24/\Omega_L+28\Omega_l/\Omega_d\Omega_L -8\Omega_l/\Omega_d{}^2+8\Omega_l{}^2/\Omega_d{}^2\Omega_L]$$

$$p_2=(1/\Omega_l)\,[-72\Omega_l/\Omega_d\Omega_L-22\Omega_l/\Omega_d{}^2-62\Omega_l{}^2/\Omega_d{}^2\Omega_L -12\Omega_l{}^2/\Omega_d{}^3-12\Omega_l{}^3/\Omega_d{}^3\Omega_L]$$

$$q_2=(1/\Omega_l)\,[12/\Omega_d-84\Omega_l/\Omega_d\Omega_L-8\Omega_l/\Omega_d{}^2-76\Omega_l{}^2/\Omega_d{}^2\Omega_L -8\Omega_l{}^2/\Omega_d{}^3-16\Omega_l{}^3/\Omega_d{}^3\Omega_L]$$

$$r_2=[64\Omega_l/\Omega_d{}^3+240/\Omega_d\Omega_L-72/\Omega_l\Omega_L+192\Omega_l/\Omega_d{}^2\Omega_L +32\Omega_l{}^2/\Omega_d{}^3\Omega_L+48/\Omega_l\Omega_d+144/\Omega_d{}^2]$$

$$s_2=[128\Omega_l/\Omega_d{}^3+72\Omega_l/\Omega_d{}^2\Omega_L+40\Omega_l{}^2/\Omega_d{}^3\Omega_L +72/\Omega_d{}^2+48\Omega_l{}^2/\Omega_d{}^4]$$

When the tension coefficients in the members are written down in the general form found, they are too cumbersome for any deductions to be drawn. Values have therefore been given to Ω_L, Ω_l and Ω_d which enable the results for a special case to be appreciated more readily.

The values assumed are

$$1/\Omega_L=1,\quad 1/\Omega_l=10 \text{ and } 1/\Omega_d=200$$

which correspond to a structure having the area of the keel ten times and the area of a diagonal bracing member approximately 1/9th that of the area of a longitudinal, when the ratio of lengths of diagonal to longitudinal is 4/3.

The results are given below in detail, in terms of N the number of bays:—

$$(R_1-R_4)=\{1/(N^4+25.2547\,N^2+136.5474)\}\,[(P_1/l)\,(0.7500\,N^3+7.5001\,N) +(P_2/l)\,(1.5000\,N^3+19.3655\,N)]$$

$$(R_2-R_4)=\{1/(N^4+25.2547\,N^2+136.5474)\}\,[(P_1/l)\,(1.5000\,N^3+10.8769\,N) +(P_2/l)\,(1.5000\,N^3+7.6024\,N)]$$

$$a_1=0.00001\,(R_1-R_4)-0.0122\,(R_2-R_4)$$
$$a_2=-0.0005\,(R_1-R_4)+0.0239\,(R_2-R_4)$$
$$a_3=0.0218\,(R_1-R_4)+0.0091\,(R_2-R_4)$$
$$(a_1-a_1)=-\tfrac{1}{2}\,(R_1-R_4)+\tfrac{1}{2}\,(R_2-R_4)+0.0243\,P_1/l+0.0243\,P_2/l$$
$$(a_2-a_2)=\tfrac{1}{2}\,(R_1-R_4)-0.0472\,P_1/l-0.0467\,P_2/l$$
$$(a_3-a_3)=-\tfrac{1}{2}\,(R_1-R_4)-\tfrac{1}{2}\,(R_2-R_4)-0.0401\,P_1/l-0.0619\,P_2/l$$

When the structure is two bays long ($N=2$), the tension coefficients in the members are those given in Table 1; p_1 and p_2 are written for P_1/l and P_2/l respectively.

TABLE 1.

A_1B_1	$+0.9042\,p_1+0.0466\,p_2$	B_1C_1	$+0.8100\,p_1+0.2374\,p_2$
A_2B_2	$+0.0050\,p_1+0.8747\,p_2$	B_2C_2	$-0.0307\,p_1+0.5794\,p_2$
A_3B_3	$-1.7695\,p_1-2.6453\,p_2$	B_3C_3	$-1.4832\,p_1-2.1534\,p_2$
A_4B_4	$+2.8584\,p_1+3.8060\,p_2$	B_4C_4	$+2.4146\,p_1+3.1700\,p_2$
A_1B_2	$+0.0479\,p_1-0.0233\,p_2$	B_1C_2	$+0.0447\,p_1-0.0259\,p_2$
A_2B_1	$-0.0024\,p_1+0.0695\,p_2$	B_2C_1	$-0.0056\,p_1+0.0669\,p_2$
A_2B_3	$-0.0026\,p_1+0.0558\,p_2$	B_2C_3	$+0.0038\,p_1+0.0608\,p_2$
A_3B_2	$-0.0854\,p_1-0.1443\,p_2$	B_3C_2	$-0.0790\,p_1-0.1393\,p_2$
A_3B_4	$-0.1451\,p_1-0.2104\,p_2$	B_3C_4	$-0.1391\,p_1-0.1998\,p_2$
A_4B_3	$+0.0708\,p_1+0.0970\,p_2$	B_4C_3	$+0.0768\,p_1+0.1076\,p_2$

When the structure is four bays long ($N=4$), the tension coefficients are given in Table 2.

TABLE 2.

A_1B_1	$+0.8782\,p_1+0.0142\,p_2$	C_1D_1	$+0.5862\,p_1+0.2654\,p_2$
A_2B_2	$+0.0104\,p_1+0.8822\,p_2$	C_2D_2	$-0.0394\,p_1+0.3212\,p_2$
A_3B_3	$-1.7351\,p_1-2.6042\,p_2$	C_3D_3	$-1.0249\,p_1-1.4560\,p_2$
A_4B_4	$+2.7998\,p_1+3.7352\,p_2$	C_4D_4	$+1.6782\,p_1+2.1800\,p_2$
A_1B_2	$+0.0609\,p_1-0.0071\,p_2$	C_1D_2	$+0.0525\,p_1-0.0147\,p_2$
A_2B_1	$-0.0163\,p_1+0.0519\,p_2$	C_2D_1	$-0.0247\,p_1+0.0443\,p_2$
A_2B_3	$+0.0059\,p_1+0.0659\,p_2$	C_2D_3	$+0.0227\,p_1+0.0811\,p_2$
A_3B_2	$-0.0920\,p_1-0.1518\,p_2$	C_3D_2	$-0.0752\,p_1-0.1366\,p_2$
A_3B_4	$-0.1729\,p_1-0.2440\,p_2$	C_3D_4	$-0.1581\,p_1-0.2192\,p_2$
A_4B_3	$+0.1001\,p_1+0.1324\,p_2$	C_4D_3	$+0.1149\,p_1+0.1572\,p_2$
B_1C_1	$+0.7322\,p_1+0.1398\,p_2$	D_1E_1	$+0.4402\,p_1+0.3910\,p_2$
B_2C_2	$-0.0145\,p_1+0.6017\,p_2$	D_2E_2	$-0.0643\,p_1+0.0407\,p_2$
B_3C_3	$-1.3800\,p_1-2.0301\,p_2$	D_3E_3	$-0.6698\,p_1-0.8819\,p_2$
B_4C_4	$+2.2390\,p_1+2.9576\,p_2$	D_4E_4	$+1.1174\,p_1+1.4024\,p_2$
B_1C_2	$+0.0567\,p_1-0.0109\,p_2$	D_1E_2	$+0.0483\,p_1-0.0185\,p_2$
B_2C_1	$-0.0205\,p_1+0.0481\,p_2$	D_2E_1	$-0.0289\,p_1+0.0405\,p_2$
B_2C_3	$+0.0143\,p_1+0.0735\,p_2$	D_2E_3	$+0.0311\,p_1+0.0887\,p_2$
B_3C_2	$-0.0836\,p_1-0.1442\,p_2$	D_3E_2	$-0.0668\,p_1-0.1290\,p_2$
B_3C_4	$-0.1655\,p_1-0.2316\,p_2$	D_3E_4	$-0.1507\,p_1-0.2068\,p_2$
B_4C_3	$+0.1075\,p_1+0.1448\,p_2$	D_4E_3	$+0.1223\,p_1+0.1696\,p_2$

The external loads P_1 and P_2 are independent of one another, therefore any relation may be assumed between them.

Tables 3, 4, give the tension coefficients for the cases $N=2$ and $N=4$, when $P_2=P_1$.

TABLE 3.

A_1B_1	$+0.9508\,p_1$	B_1C_1	$+1.0474\,p_1$
A_2B_2	$+0.8797\,p_1$	B_2C_2	$+0.5487\,p_1$
A_3B_3	$-4.4148\,p_1$	B_3C_3	$-3.6366\,p_1$
A_4B_4	$+6.6644\,p_1$	B_4C_4	$+5.5846\,p_1$
A_1B_2	$+0.0246\,p_1$	B_1C_2	$+0.0188\,p_1$
A_2B_1	$+0.0671\,p_1$	B_2C_1	$+0.0613\,p_1$
A_2B_3	$+0.0532\,p_1$	B_2C_3	$+0.0646\,p_1$
A_3B_2	$-0.2297\,p_1$	B_3C_2	$-0.2183\,p_1$
A_3B_4	$-0.3555\,p_1$	B_3C_4	$-0.3389\,p_1$
A_4B_3	$+0.1678\,p_1$	B_4C_3	$+0.1844\,p_1$

TABLE 4.

A_1B_1	$+0.8924\,p_1$	B_1C_1	$+0.8720\,p_1$	C_1D_1	$+0.8516\,p_1$	D_1E_1	$+0.8312\,p_1$
A_2B_2	$+0.8926\,p_1$	B_2C_2	$+0.5872\,p_1$	C_2D_2	$+0.2818\,p_1$	D_2E_2	$-0.0236\,p_1$
A_3B_3	$-4.3393\,p_1$	B_3C_3	$-3.4101\,p_1$	C_3D_3	$-2.4809\,p_1$	D_3E_3	$-1.5517\,p_1$
A_4B_4	$+6.5350\,p_1$	B_4C_4	$+5.1966\,p_1$	C_4D_4	$+3.8582\,p_1$	D_4E_4	$+2.5198\,p_1$
A_1B_2	$+0.0538\,p_1$	B_1C_2	$+0.0458\,p_1$	C_1D_2	$+0.0378\,p_1$	D_1E_2	$+0.0298\,p_1$
A_2B_1	$+0.0356\,p_1$	B_2C_1	$+0.0276\,p_1$	C_2D_1	$+0.0196\,p_1$	D_2E_1	$+0.0116\,p_1$
A_2B_3	$+0.0718\,p_1$	B_2C_3	$+0.0878\,p_1$	C_2D_3	$+0.1038\,p_1$	D_2E_3	$+0.1198\,p_1$
A_3B_2	$-0.2438\,p_1$	B_3C_2	$-0.2278\,p_1$	C_3D_2	$-0.2118\,p_1$	D_3E_2	$-0.1958\,p_1$
A_3B_4	$-0.4169\,p_1$	B_3C_4	$-0.3971\,p_1$	C_3D_4	$-0.3773\,p_1$	D_3E_4	$-0.3575\,p_1$
A_4B_3	$+0.2325\,p_1$	B_4C_3	$+0.2523\,p_1$	C_4D_3	$+0.2721\,p_1$	D_4E_3	$+0.2919\,p_1$

It can be seen from these tables that the loads in the members of types A_2B_3 and A_4B_3 increase from the free end where the external load is applied; the loads in type A_1B_1 increase when $N=2$, but decrease when $N=4$. It will be seen from Table 5, which is obtained by making $P_2=10\,P_1$, that A_1B_1 is now made to increase for the case $N=4$; A_2B_3 and A_4B_3 increase as before, and in addition A_1B_2 does so for this relation between the external loads.

TABLE 5.

$A_1B_1 + 1.0202\,p_1$	$B_1C_1 + 2.1302\,p_1$	$C_1D_1 + 3.2402\,p_1$	$D_1E_1 + 4.3502\,p_1$
$A_2B_2 + 8.8324\,p_1$	$B_2C_2 + 6.0025\,p_1$	$C_2D_2 + 3.1726\,p_1$	$D_2E_2 + 0.3427\,p_1$
$A_3B_3 - 27.7771\,p_1$	$B_3C_3 - 21.6810\,p_1$	$C_3D_3 - 15.5849\,p_1$	$D_3E_3 - 9.4888\,p_1$
$A_4B_4 + 40.1518\,p_1$	$B_4C_4 + 31.8150\,p_1$	$C_4D_4 + 23.4782\,p_1$	$D_4E_4 + 15.1414\,p_1$
$A_1B_2 - 0.0101\,p_1$	$B_1C_2 - 0.0523\,p_1$	$C_1D_2 - 0.0945\,p_1$	$D_1E_2 - 0.1367\,p_1$
$A_2B_1 + 0.5027\,p_1$	$B_2C_1 + 0.4605\,p_1$	$C_2D_1 + 0.4183\,p_1$	$D_2E_1 + 0.3761\,p_1$
$A_2B_3 + 0.6649\,p_1$	$B_2C_3 + 0.7493\,p_1$	$C_2D_3 + 0.8337\,p_1$	$D_2E_3 + 0.9181\,p_1$
$A_3B_2 - 1.6100\,p_1$	$B_3C_2 - 1.5256\,p_1$	$C_3D_2 - 1.4412\,p_1$	$D_3E_2 - 1.3568\,p_1$
$A_3B_4 - 2.6129\,p_1$	$B_3C_4 - 2.4815\,p_1$	$C_3D_4 - 2.3501\,p_1$	$D_3E_4 - 2.2187\,p_1$
$A_4B_3 + 1.4241\,p_1$	$B_4C_3 + 1.5555\,p_1$	$C_4D_3 + 1.6869\,p_1$	$D_4E_3 + 1.8183\,p_1$

These results show that it is possible, therefore, in this structure for the loads in certain types of members to increase as they leave the neighbourhood of an applied system, having no resultant force and no resultant couple. This appears to be a contradiction of the principle of St. Venant, which states that the strains in a body, under such a system as that considered, are of negligible magnitude at distances from the part where the external loads are applied, which are large compared with the linear dimensions of that part.

The weight of evidence in favour of the reliability of St. Venant's principle, since the whole theory of elasticity is based on it, is too great to admit of contradiction; but the case in question is not wholly comparable with that of a solid body. While the principle does hold for a very highly redundant structure, as a solid body, it may not do so in exactly the terms stated for one much less redundant as this tubular structure.

Mr. R. V. Southwell, F.R.S., in dealing with this matter,* has stated the principle in the following form:—

"Forces applied at one part of an elastic structure will induce stresses which, except in a region close to that part, will depend almost entirely upon their resultant action and very little upon their distribution." And continues, "the theorem just stated may be regarded as an immediate consequence of the tendency towards a minimum storage of strain energy. Let us consider the case of a long girder, or braced framework, which is loaded at its two ends by forces applied in any given way, and let us employ the symbol 'A' to denote those regions which immediately adjoin the parts at which the forces are applied and the symbol 'B' for the remainder. The conditions of equilibrium require merely that the resultant action transmitted by 'B' shall have a definite value; but the conditions of continuity (or of compatibility of strains) will not be satisfied unless the total increase of the strain energy stored in 'A' and 'B' has its minimum value. Evidently, then, the equilibrium configuration may be regarded as in the nature of a compromise between the requirements of 'A' and 'B.' To reduce to a minimum the strain energy stored in 'A,' the reactions between 'A' and 'B' would require to distribute themselves in a manner which will depend upon the distribution of the forces applied to 'A'; the requirements of 'B,' on the other hand, will not vary, since there must be

* "On Castigliano's Theorem of Least Work, and the Principle of St. Venant," *Philosophical Magazine*, January, 1923.

some definite distribution of stresses in an otherwise unloaded body which will entail the minimum storage of strain energy in transmitting a given resultant action.

" Thus, in the process of adjustment which results in the actual distribution corresponding to equilibrium, we may picture a contest between the unloaded portions which strive always after ' standardisation ' and the loaded portions which demand a particular solution for every specified distribution of the forces acting on them. As we pass from the regions of application of load through successive sections of the unloaded portion ' B,' there will be a steady tendency for the claims of standardisation to prevail. The theorem stated (above) is an immediate deduction and we may work back from this, by principle of superposition, to St. Venant's principle."

If, in the tubular structure four bays long which has been stressed for a symmetrical external load system of P_1, $10\,P_1$, $-32\,P_1$, $43\,P_1$, $-32\,P_1$ and $10\,P_1$, acting at A_1, A_2, A_3, A_4, A_5 and A_6 respectively, the first bay is called, according to Mr. Southwell's notation, " A," and the remainder " B "; then the distribution of resultant action between " A " and " B," to make the energy stored in them a minimum, can be found from Table 5, and is $2.0256\,P_1$, $7.2123\,P_1$, $-25.6881\,P_1$ and $34.9260\,P_1$ at B_1, B_2, B_3 and B_4.

The distribution required at these joints to make the energy stored in " A " a minimum can be found from the general expressions by making $N=1$. It is found to be $2.3940\,P_1$, $7.9710\,P_1$, $-28.7010\,P_1$ and $39.0660\,P_1$.

" B," considered alone, requires zero load at each joint B_1, B_2, etc., and therefore, comparing the three distributions, it can be seen that the actual distribution between " A " and " B " is tending towards the standardisation demanded by " B."

The loads in the members thus satisfy the conditions of distribution as stated by Mr. Southwell, and yet do not all decrease as the distance from the loaded region increases.

If the portions " A " and " B " of a structure were separated by a portion " C," where " A " and " B " considered alone were similar simple structures, then it is evident that if " C " was redundant it would be possible, by selecting the redundancy, to ensure that one member at least of " B " would receive a larger load than that in the corresponding member of " A," due to the external system (with no resultant actions) applied at " A." The question thus appears to be solely one of relative redundancy.

The energy stored in the tubular structure may be examined. The energy in any bay is the sum of $p^2/2\Omega$ for all the members in that bay.

When $N=2$, $P_1=P_2$, Table 3.

Energy in bay AB $= 268.49\,p^2$
,, ,, BC $= 197.02\,p^2$

When $N=4$, $P_1=P_2$, Table 4.

Energy in bay AB $= 280.65\,p^2$
,, ,, BC $= 193.50\,p^2$
,, ,, CD $= 127.70\,p^2$
,, ,, DE $= 84.07\,p^2$

In each case the energy stored in the bays decreases as the region of applied load is left.

Therefore it appears probable, from the evidence obtained, that when a system of forces, giving no resultant axial force and no resultant couple, is applied to one part of a redundant structure, the energy stored in one section is less than that stored in a corresponding section of the structure, which is

nearer the applied load system; but the loads in certain corresponding members in the two sections may increase in the direction the energy decreases.

It is unfortunate that the general results obtained are too cumbersome to allow more general deductions to be drawn, and some more elegant method must be used to decide what degree of redundancy is necessary to ensure the structure behaving, in this respect, as a solid body. Until this is done it is impossible to forecast the distribution of such stress in an actual airship hull; but the danger of large secondary stresses being set up by a pyramidal nose will be decreased if the base of the pyramid is made, as far as possible, to remain plane; that is to say, if the transverse frame to which the nose piece is attached is designed to resist distortion out of its plane.

Part II.

Secondary Stresses Due to the Continuity of the Longitudinals

When the stresses in a structure have been calculated on the assumption that all members are free to turn at the joints, it is possible to calculate the additional stresses due to the actual rigidity of the joints by an exact method given by Manderla. In applying this method considerable labour is involved, and it is usual in most branches of structural engineering to make use of an approximate method in which the bending moment in the member, due to the primary stress acting with an arm equal to the deflection of the member, is neglected. The work involved even in this approximate method is very heavy for the simplest two-dimensional frame. For a space frame, such as an airship hull, it would be quite prohibitive.

In an airship hull the longitudinal girders are usually continuous over the transverse frames, and it is this continuity which has most effect on the distribution of stress. That this redistribution of load in the members may be a serious matter has been shown by experiments carried out on hulls of the Zeppelin type, the results of which are given by Mr. E. H. Lewitt.* In these experiments a parallel portion of the hull was held at one main frame and loads were applied over several bays. The tensions in diagonal wires were measured by tautness meters before and after loading. It was found in every case that the shear taken by the diagonal wires was only a part of the total. The shear taken by the longitudinals acting as cantilevers was found to be nearly equal in certain cases (of which R.38 was one) to that taken by the diagonal wires. It is essential therefore that, in considering a design in which the secondary stresses have not been eliminated by articulating the longitudinals or by other means, methods must be found which enable results to be obtained in a reasonable time by the design staff.

A method for determining the secondary stresses in two-dimensional frames has been given by Professor Z. Bazant, of Prague, a summary of which is given in Appendix A of this paper and which, by a slight extension, may be applied to the problem of the continuous longitudinal.

By this method one class of members in the structure, for instance, the lower chord members in a Warren girder, is taken to be continuous while all the other members are pin-jointed to it and to one another. Considering these rigidly connected members as a continuous beam, the joints, where other members are pinned to it, being supports, it is possible, from a knowledge of the primary stresses in the structure, to find the deflection of any joint or support in it with respect to the joints on either side. The equation in terms of the three fixing moments at these joints may be written down in terms of this deflection as for a continuous beam. The relative deflection of each joint in turn and from it

* Page 56, *The Rigid Airship*. Sir Isaac Pitman & Sons.

the corresponding equation may be written down. Enough equations are then forthcoming for all the moments at the joints to be found, as in the Theorem of Three Moments. These moments are due to the rigidity of the joints and enable the secondary fibre stresses to be estimated.

Such an assumption of continuous longitudinals to which all other members are pinned is very fair in the case of an airship.

The effect of the continuity at each joint, where the longitudinal crosses a transverse frame, may be expressed as:—

(1) A moment in the plane containing the longitudinal and the axis of the ship.

(2) A moment with its axis the radius of the transverse frame to that joint.

(3) A moment with its axis lying in the plane containing the longitudinal and the axis of the ship, perpendicular to the radius of the transverse frame to that joint.

To determine the moment, type (1), denoted by M_r, consider a portion of

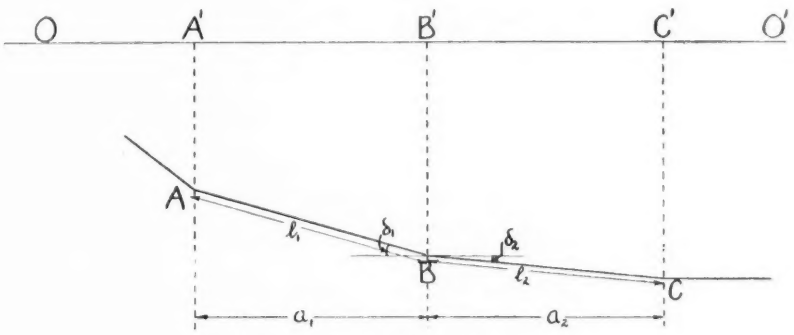


FIG. 1, PART II.

the longitudinal (Fig. 1, Pt. II.) where OO' represents the axis of the ship, and AA', BB', CC' are the radii of the transverse frames at A, B and C in the plane containing the longitudinal and the axis OO'.

δ_1 and δ_2 are the angles which the parts AB and BC of the longitudinal make with the axis.

a_1 and a_2 are the lengths of $A'B'$ and $B'C'$.

From a knowledge of the primary stresses the relative deflection y_B, in the radial direction BB', of the point B with respect to A and C may be found, and as in Appendix A the resulting equation is

$$M_{rA}(a_1/\cos\delta_1)+2M_{rB}\{(a_1/\cos\delta_1)+(a_2/\cos\delta_2)(I_1/I_2)\}$$
$$+M_{rC}(a_2/\cos\delta_2)(I_1/I_2)=(6EI_1/a_1a_2)(a_1+a_2)y_B$$

where M_{rA}, M_{rB} and M_{rC} are the fixing moments at A, B and C; I_1 and I_2 are the moments of inertia of the cross-sections of AB and BC respectively.

From a consideration of each joint in the longitudinal in this way, an equation is obtained, providing as many equations as there are unknown moments M_{rA}, M_{rB}, etc., which can therefore be found.

The equations found by Professor Bazant cannot be used to determine the moments of types (2) and (3) denoted by M_t and T respectively, but somewhat similar expressions may be found for them.

The relative deflection x_B, of the point B with respect to A and C, in a direction at right angles to the radius BB', that is, tangential to the transverse frame, may be found by the same methods used to determine y_B.

After loading, the projections of AB and BC on the plane of the couple M_{tB} make angles x_B/a_1 and $-x_B/a_2$ with the horizontal.

If the internal work of the part AB is called W_1 and that of the part BC W_2, then by Castigliano's theorem of the differential coefficients of internal work

$$+x_B(1/a_2+1/a_1)=\partial W_1/\partial M_{tB}+\partial W_2/\partial M_{tB}$$

In the span AB consider a section G at a distance "x" from A, then

$$M_G=\{(l_1-x)/l_1\}(M_{tA}\cos\delta_1-T_A\sin\delta_1)+(x/l_1)(M_{tB}\cos\delta_1-T_B\sin\delta_1)$$

$$T_G=M_{tA}\sin\delta_1+T_A\cos\delta_1$$

where M_G and T_G are the bending moment and torque respectively at section G.

Neglecting shear

$$W_1=\tfrac{1}{2}\int_0^{l_1}(M_G^2/EI_1)\,dx+\tfrac{1}{2}\int_0^{l_1}(T_G^2/CJ_1)\,dx$$

where CJ_1 defines the resistance of the member to torque, then

$$\partial W_1/\partial M_{tB}=(1/l_1)\int_0^{l_1}(M_G/EI_1)\,.\,x\cos\delta_1\,.\,dx$$

$$=(l_1\cos\delta_1/EI_1)\left[\left(\tfrac{1}{6}\right)(M_{tA}\cos\delta_1-T_A\sin\delta_1)+\left(\tfrac{1}{3}\right)(M_{tB}\cos\delta_1-T_B\sin\delta_1)\right]$$

Similarly

$$\partial W_2/\partial M_{tB}=(l_2\cos\delta_2/EI_2)\left[\left(\tfrac{1}{6}\right)(M_{tC}\cos\delta_2-T_C\sin\delta_2)+\left(\tfrac{1}{3}\right)(M_{tB}\cos\delta_2-T_B\sin\delta_2)\right]$$

Therefore

$$+6x_B(1/a_1+1/a_2)=l_1\cos\delta_1\{(M_{tA}\cos\delta_1-T_A\sin\delta_1)+2(M_{tB}\cos\delta_1-T_B\sin\delta_1)\}(1/EI_1)$$
$$+l_2\cos\delta_2\{(M_{tC}\cos\delta_2-T_C\sin\delta_2)+2(M_{tB}\cos\delta_2-T_B\sin\delta_2)\}(1/EI_2)$$

Also from the continuity of the girder at B

$$\partial W_1/\partial T_B=-\partial W_2/\partial T_B$$

or

$$l_1\sin\delta_1\{(M_{tA}\cos\delta_1-T_A\sin\delta_1)+2(M_{tB}\cos\delta_1-T_B\sin\delta_1)\}(1/EI_1)$$
$$+l_2\sin\delta_2\{(M_{tC}\cos\delta_2-T_C\sin\delta_2)+2(M_{tB}\cos\delta_2-T_B\sin\delta_2)\}(1/EI_2)=0$$

Two equations, such as these, will be forthcoming from a consideration of each joint, and the unknown couples may be found from them. The solving of these equations for the whole longitudinal may be troublesome, but where there is a parallel portion in a ship the expressions are simplified, since $\delta_1=\delta_2=0$ and T vanishes.

Leaving only one equation

$$(a_1/I_1)M_{tA}+2(a_1/I_1+a_2/I_2)M_{tB}++(a_2/I_2)M_{tC}=6E(1/a_1+1/a_2)x_B$$

which is identical with that obtained for the radial deflection.

In streamlined ships the point B will be so nearly on the line joining A and C, over the greater length of the ship, that in a first approximation the term T may be neglected and the simplified expression used.

It is difficult to estimate with any degree of accuracy the amount of work any method will entail until it has been applied.

It is impossible, in the compass of this paper, to find the secondary stresses in an actual airship hull, but a simple case has been analysed, which does not entail a great amount of work and yet which demonstrates all the necessary steps and shows exactly what labour would be involved in practice.

The structure considered (Fig 2, Pt. II.) is similar to that used in Part I. It is hexagonal in cross-section, the longitudinals are continuous over the second and third frames B and C, but are pinned at the free end A and to the wall D.

It is three bays in length, the panels are counterbraced and all transverse frames are radially braced.

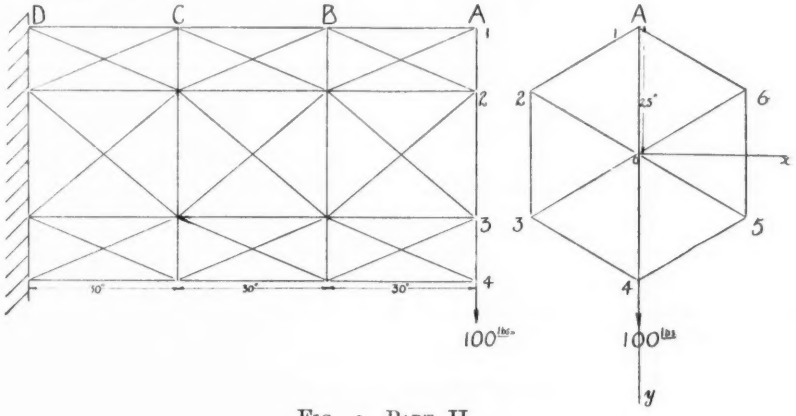


FIG. 2, PART II.

The radius of each transverse frame is 25 inches and they are 30 inches apart. The keel member is a solid $\frac{7}{8}$in. rod, all other longitudinals and all transverse members are made up of tube $\frac{7}{8}$in. O.D. × 22g. The diagonal bracing members are 4 B.A. wires, initially tensioned so as to be always operative. The radial bracing is assumed to be of large cross-section compared with the transverse members. The material used throughout is steel, $E = 13{,}600$ tons/sq. in.

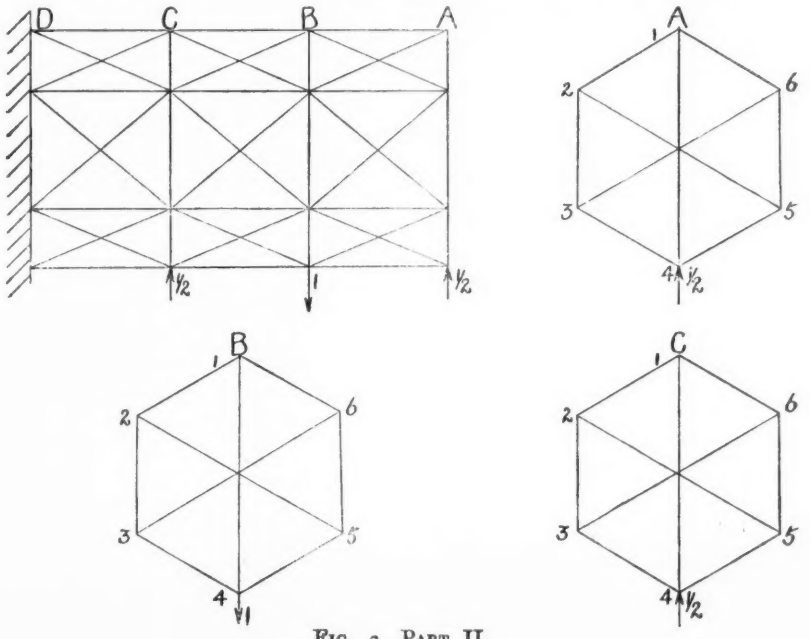


FIG. 3, PART II.

A load of 100 lbs. acts at A, as shown in Fig. 2, Pt. II. The primary loads (when all members are assumed pin-ended) due to this external load and to the auxiliary systems which are applied, are determined by the methods of R. and M. 800, App. 3. Details of these calculations are given in Appendix B of this paper.

The primary loads in the members due to the external load of 100 lbs. are given in Table 6, column 3.

Longitudinal 4

First consider longitudinal 4 ($A_4B_4C_4D_4$).

From symmetry it can be seen that the only secondary moments in it will lie in the plane containing the longitudinal and the axis of the tube, that is type 1 above.

Apply an auxiliary system as shown in Fig. 3, Pt. II. The loads (σ_1) due to this system are given in Table 7, column 1. Considering only the members in bays AB and BC

$$\Sigma\sigma_1 \,.\, E \,.\, \Delta l = E y_B$$

Δl being the stretch in the member due to its primary load.

TABLE 6.

Member	Length (inches)	Area of Cross-Sect. (sq. in.)	Load (lb.)	$E\Delta l$
A_1A_2	25	0.0668	−23.8	− 8,910
A_2A_3	,,	,,	− 2.9	− 1,086
A_3A_4	,,	,,	+26.7	+ 10,000
A_4A_5	,,	,,	+26.7	+ 10,000
A_5A_6	,,	,,	− 2.9	− 1,086
A_6A_1	,,	,,	−23.8	− 8,910
A_1B_1	30	,,	+13.4	+ 6,050
A_2B_2	,,	,,	+ 9.1	+ 4,090
A_3B_3	,,	,,	+ 0.4	+ 180
A_4B_4	,,	0.6010	−35.6	− 1,778
A_5B_5	,,	0.0668	+ 0.4	+ 180
A_6B_6	,,	,,	+ 9.1	+ 4,090
A_1B_2	39.06	0.0085	− 8.7	− 40,100
A_2B_1	,,	,,	+10.3	+ 47,500
A_2B_3	,,	,,	−22.2	−102,000
A_3B_2	,,	,,	+22.8	+105,000
A_3B_4	,,	,,	−23.3	−107,000
A_4B_3	,,	,,	+23.1	+106,200
A_4B_5	,,	,,	+23.1	+106,200
A_5B_4	,,	,,	−23.3	−107,000
A_5B_6	,,	,,	+22.8	+105,000
A_6B_5	,,	,,	−22.2	−102,000
A_6B_1	,,	,,	+10.3	+ 47,500
A_1B_6	,,	,,	− 8.7	− 40,100
B_1B_2	25	0.0668	− 1.17	− 438
B_2B_3	,,	,,	+ 0.03	+ 11
B_3B_4	,,	,,	+ 1.1	+ 424
B_4B_5	,,	,,	+ 1.1	+ 424
B_5B_6	,,	,,	+ 0.03	+ 11
B_6B_1	,,	,,	− 1.1	− 438
B_1C_1	30	,,	+40.3	+ 18,100
B_2C_2	,,	,,	+27.3	+ 12,250
B_3C_3	,,	,,	+ 1.2	+ 539

Member	Length (inches)	Area of Cross-Sect. (sq. in.)	Load (lb.)	$E\Delta l$
B_4C_4	,,	0.6010	−106.8	− 5,310
B_5C_5	,,	0.0668	+ 1.2	+ 539
B_6C_6	,,	,,	+27.3	+ 12,250
B_1C_2	39.06	0.0085	− 7.1	− 32,800
B_2C_1	,,	,,	+11.9	+ 54,900
B_2C_3	,,	,,	−21.5	− 98,900
B_3C_2	,,	,,	+23.5	+108,000
B_3C_4	,,	,,	−23.5	−108,250
B_4C_3	,,	,,	+22.9	+105,250
B_4C_5	,,	,,	+22.9	+105,250
B_5C_4	,,	,,	−23.5	−108,250
B_5C_6	,,	,,	+23.5	+108,000
B_6C_5	,,	,,	−21.5	− 98,900
B_6C_1	,,	,,	+11.9	+ 54,900
B_1C_6	,,	,,	− 7.1	− 32,800
C_1C_2	25	0.0668	− 2.3	− 876
C_2C_3	,,	,,	+ 0.06	+ 22
C_3C_4	,,	,,	+ 2.2	+ 848
C_4C_5	,,	,,	+ 2.2	+ 848
C_5C_6	,,	,,	+ 0.06	+ 22
C_6C_1	,,	,,	− 2.3	− 876
C_1D_1	30	,,	+67.2	+ 30,200
C_2D_2	,,	,,	+45.5	+ 20,420
C_3D_3	,,	,,	+ 2.0	+ 900
C_4D_4	,,	0.6010	−178.1	− 8,890
C_5D_5	,,	0.0668	+ 2.0	+ 900
C_6D_6	,,	,,	+45.5	+ 20,420
C_1D_2	39.06	0.0085	− 5.5	− 25,450
C_2D_1	,,	,,	+13.5	+ 62,200
C_2D_3	,,	,,	−20.9	− 96,000
C_3D_2	,,	,,	+24.1	+110,800
C_3D_4	,,	,,	−23.7	−109,000
C_4D_3	,,	,,	+22.7	+104,250
C_4D_5	,,	,,	+22.7	+104,250
C_5D_4	,,	,,	−23.7	−109,000
C_5D_6	,,	,,	+24.1	+110,800
C_6D_5	,,	,,	−20.9	− 96,000
C_6D_1	,,	,,	+13.5	+ 62,200
C_1D_6	,,	,,	− 5.5	− 25,450

The values of $E\Delta l$ are obtained from Table 6, the summation of $\sigma_1 E\Delta l$ is given in Table 7, giving Ey_B.

Since the longitudinal is pinned at A_4, $M_{rA4}=0$ and

$$4M_{rB4}+M_{rC4}=(-12\times I_L\times 9{,}477)/(30)^2$$

or

$$4M_{rB4}+M_{rC4}=-126.2\,I_L \quad . \quad . \quad . \quad . \quad (1)$$

Now move the auxiliary system to the joints $B_4C_4D_4$, as shown in Fig. 4, Pt. II. It will give loads as shown in Table 8 and Ey_C may be evaluated, Table 8 giving

$$M_{rB4}+4M_{rC4}=(-12\times I_L\times 9{,}899)/(30)^2, \text{ since } M_{rD4}=0$$

or

$$M_{rB4}+4M_{rC4}=-132.0\,I_L \quad . \quad . \quad . \quad . \quad (2)$$

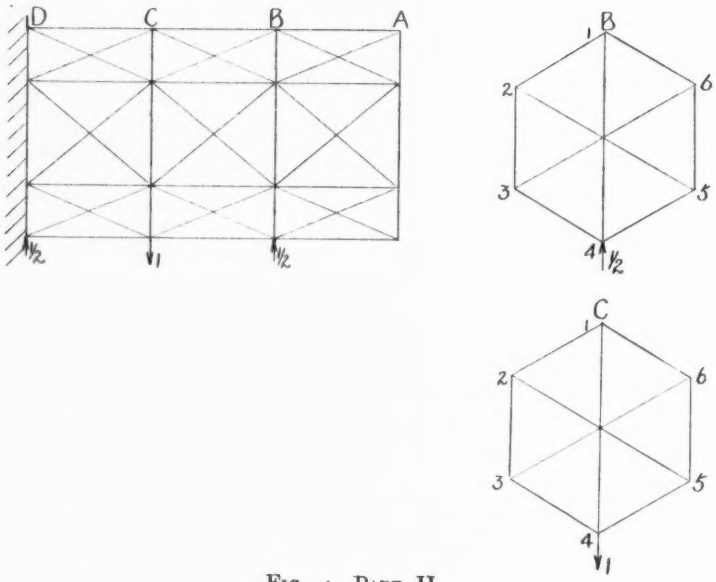


FIG. 4, PART II.

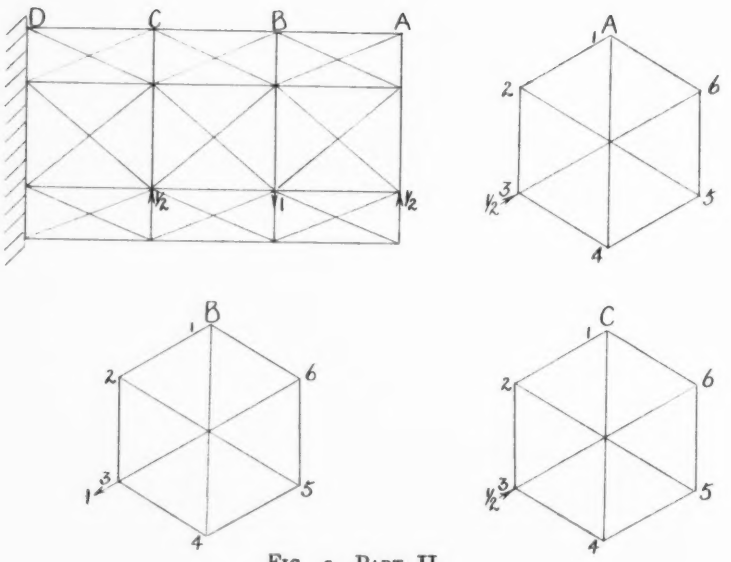


FIG. 5, PART II.

From (1) and (2)

$$\left.\begin{aligned} M_{rB4} &= -24.85\, I_L \\ M_{rC4} &= -26.78\, I_L \end{aligned}\right\}$$

TABLE 7.

LONGITUDINAL 4.

RADIAL AUXILIARY SYSTEM AT *ABC*.

Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$	Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$
A_1A_2	+0.1190	− 8,910	− 1,060	B_1B_2	−0.2320	− 438	+ 102
A_2A_3	+0.0140	− 1,086	− 15	B_2B_3	−0.0290	+ 11	− 0
A_3A_4	−0.1330	+ 10,000	− 1,330	B_3B_4	+0.2610	+ 424	+ 110
A_4A_5	−0.1330	+ 10,000	− 1,330	B_4B_5	+0.2610	+ 424	+ 110
A_5A_6	+0.0140	− 1,086	− 15	B_5B_6	−0.0290	+ 11	− 0
A_6A_1	+0.1190	− 8,910	− 1,060	B_6B_1	−0.2320	− 438	+ 102
A_1B_1	−0.0673	+ 6,050	− 406	B_1C_1	−0.0673	+ 18,100	− 1,220
A_2B_2	−0.0455	+ 4,090	− 186	B_2C_2	−0.0455	+ 12,250	− 558
A_3B_3	−0.0020	+ 180	− 0	B_3C_3	−0.0020	+ 539	− 1
A_4B_4	+0.1780	− 1,778	− 316	B_4C_4	+0.1780	− 5,310	− 945
A_5B_5	−0.0020	+ 180	− 0	B_5C_5	−0.0020	+ 539	− 1
A_6B_6	−0.0455	+ 4,090	− 186	B_6C_6	−0.0455	+ 12,250	− 558
A_1B_2	+0.0438	− 40,100	− 1,760	B_1C_2	−0.0518	− 32,800	+ 1,700
A_2B_1	−0.0518	+ 47,500	− 2,460	B_2C_1	+0.0438	+ 54,900	+ 2,400
A_2B_3	+0.1110	− 102,000	− 11,320	B_2C_3	−0.1142	− 98,900	+ 11,280
A_3B_2	−0.1142	+ 105,000	− 12,000	B_3C_2	+0.1110	+ 108,000	+ 12,000
A_3B_4	+0.1167	− 107,000	− 12,480	B_3C_4	−0.1157	− 108,250	+ 12,500
A_4B_3	−0.1157	+ 106,200	− 12,300	B_4C_3	+0.1167	+ 105,250	+ 12,300
A_4B_5	−0.1157	+ 106,200	− 12,300	B_4C_5	+0.1167	+ 105,250	+ 12,300
A_5B_4	+0.1167	− 107,000	− 12,480	B_5C_4	−0.1157	− 108,250	+ 12,500
A_5B_6	−0.1142	+ 105,000	− 12,000	B_5C_6	+0.1110	+ 108,000	+ 12,000
A_6B_5	+0.1110	− 102,000	− 11,320	B_6C_5	−0.1142	− 98,900	+ 11,280
A_6B_1	−0.0518	+ 47,500	− 2,460	B_6C_1	+0.0438	+ 54,900	+ 2,400
A_1B_6	+0.0438	− 40,100	− 1,760	B_1C_6	−0.0518	− 32,800	+ 1,700
				C_1C_2	+0.1190	− 876	− 104
				C_2C_3	+0.0140	+ 22	+ 0
				C_3C_4	−0.1330	+ 848	− 113
				C_4C_5	−0.1330	+ 848	− 113
				C_5C_6	+0.0140	+ 22	+ 0
				C_6C_1	+0.1190	− 876	− 104

$$\sum_{B}^{A} = -110{,}544$$

$$\sum_{C}^{B} = +101{,}067$$

$$\sum_{C}^{A} = Ey_B = -9{,}477$$

$$\sum_{C}^{B} = +101{,}067$$

Longitudinal 3

In longitudinal $A_3B_3C_3D_3$ there will be secondary moments of types (1) and (2). The former may be evaluated exactly as for longitudinal 4, by applying auxiliary systems in the radial plane (Fig. 5, Pt. II.).

The full tables are not given as they are similar to those for the case above and the amount of work involved is exactly the same.

The resulting equations are

$$4M_{rB3} + M_{rC3} = -65.1\, I_1$$

and

$$M_{rB3} + 4M_{rC3} = -59.1\, I_1$$

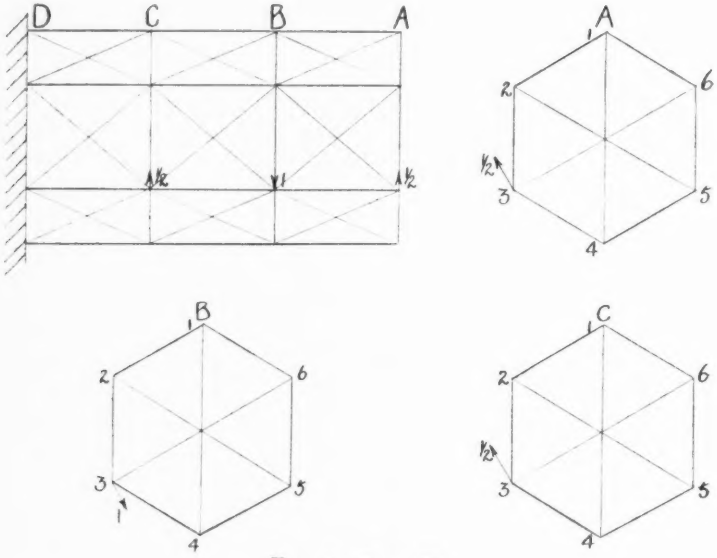


FIG. 6, PART II.

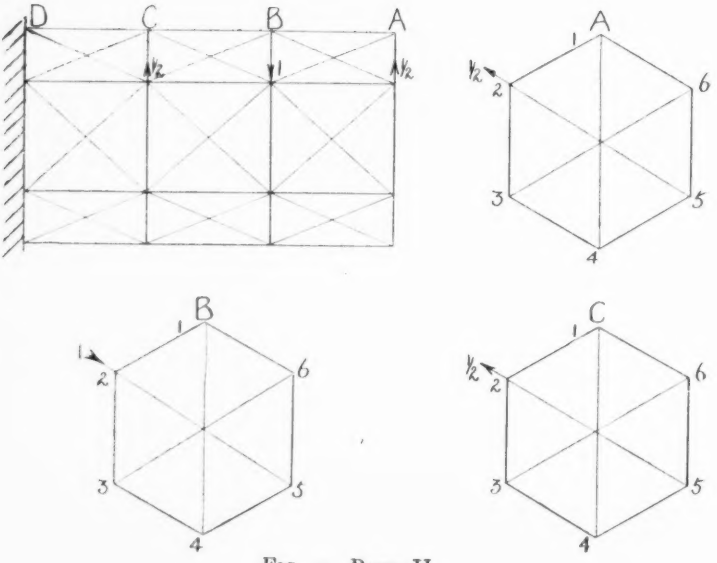


FIG. 7, PART II.

giving

$$M_{rB3} = -11.4\ I_1$$
$$M_{rC3} = -13.4\ I_1$$

For the determination of moments type (2) apply an auxiliary system as shown in Fig. 6, Pt. II., lying in the plane tangential to the transverse frames at longitudinal 3. Table 9, column 1, gives the loads due to this system and Table 9 gives the value of Ex_B, and so

$$4M_{tB3} + M_{tC3} = -2451 \times 12 \times I_1/(30)^2$$

or

$$4M_{tB3} + M_{tC3} = -32.7\ I_1 \quad . \quad . \quad . \quad . \quad (3)$$

On moving the auxiliary system to joints $B_3C_3D_3$, the resulting equation is

$$M_{tB3} + 4M_{tC3} = -111.0\ I_1 \quad . \quad . \quad . \quad . \quad (4)$$

From (3) and (4)

$$M_{tB3} = -1.3\ I_1$$
$$M_{tC3} = -27.4\ I_1$$

TABLE 8.

LONGITUDINAL 4.

RADIAL AUXILIARY SYSTEM AT *BCD*.

Member	σ_1	$E \cdot \Delta l$	$\sigma_1 \cdot E \cdot \Delta l$	Member	σ_1	$E \cdot \Delta l$	$\sigma_1 \cdot E \cdot \Delta l$
B_1B_2	+0.1190	− 438	− 52	C_1C_2	−0.2319	− 876	+ 202
B_2B_3	+0.0140	+ 11	+ 0	C_2C_3	−0.0293	+ 22	− 0
B_3B_4	−0.1330	+ 424	− 56	C_3C_4	+0.2613	+ 848	+ 221
B_4B_5	−0.1330	+ 424	− 56	C_4C_5	+0.2613	+ 848	+ 221
B_5B_6	+0.0140	+ 11	+ 0	C_5C_6	−0.0293	+ 22	− 0
B_6B_1	+0.1190	− 438	− 52	C_6C_1	−0.2319	− 876	+ 202
B_1C_1	−0.0673	+ 18,100	− 1,220	C_1D_1	−0.0673	+ 30,200	− 2,030
B_2C_2	−0.0455	+ 12,250	− 558	C_2D_2	−0.0455	+ 20,420	− 930
B_3C_3	−0.0020	+ 539	− 1	C_3D_3	−0.0020	+ 900	− 2
B_4C_4	+0.1780	− 5,310	− 945	C_4D_4	+0.1780	− 8,890	− 1,580
B_5C_5	−0.0020	+ 539	− 1	C_5D_5	−0.0020	+ 900	− 2
B_6C_6	−0.0455	+ 12,250	− 558	C_6D_6	−0.0455	+ 20,420	− 2,030
B_1C_2	+0.0438	− 32,800	− 1,438	C_1D_2	−0.0518	− 25,450	+ 1,320
B_2C_1	−0.0518	+ 54,900	− 2,840	C_2D_1	+0.0438	+ 62,200	+ 2,722
B_2C_3	+0.1110	− 98,900	− 10,980	C_2D_3	−0.1142	− 96,000	+ 10,950
B_3C_2	−0.1142	+ 108,000	− 12,350	C_3D_2	+0.1110	+ 110,800	+ 12,300
B_3C_4	+0.1167	− 108,250	− 12,620	C_3D_4	−0.1157	− 109,000	+ 12,600
B_4C_3	−0.1157	+ 105,250	− 12,190	C_4D_3	+0.1167	+ 104,250	+ 12,190
B_4C_5	−0.1157	+ 105,250	− 12,190	C_4D_5	+0.1167	+ 104,250	+ 12,190
B_5C_4	+0.1167	− 108,250	− 12,620	C_5D_4	−0.1157	− 109,000	+ 12,600
B_5C_6	−0.1142	+ 108,000	− 12,350	C_5D_6	+0.1110	+ 110,800	+ 12,300
B_6C_5	+0.1110	− 98,900	− 10,980	C_6D_5	−0.1142	− 96,000	+ 10,950
B_6C_1	−0.0518	+ 54,900	− 2,840	C_6D_1	+0.0438	+ 62,200	+ 2,722
B_1C_6	+0.0438	− 32,800	− 1,438	C_1D_6	−0.0518	− 25,450	+ 1,320

$$\sum_{C}^{B} = -108,335$$

$$\sum_{D}^{C} = +98,436$$

$$\sum_{D}^{B} = Ey_C = -9,899$$

$$\sum_{D}^{C} = +98,436$$

TABLE 9.

LONGITUDINAL 3.

TANGENTIAL AUXILIARY SYSTEM AT *ABC*.

Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$	Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$
A_1A_2	−0.0030	− 8,910	+ 27	B_1B_2	+0.020	− 438	− 9
A_2A_3	−0.1530	− 1,086	+ 166	B_2B_3	+0.314	+ 11	+ 3
A_3A_4	+0.1620	+ 10,000	+ 1,620	B_3B_4	−0.326	+ 424	− 138
A_4A_5	−0.0080	+ 10,000	− 80	B_4B_5	+0.008	+ 424	+ 3
A_5A_6	−0.0140	− 1,086	+ 15	B_5B_6	+0.020	+ 11	+ 0
A_6A_1	+0.017	− 8,910	− 152	B_6B_1	−0.038	− 438	+ 17
A_1B_1	−0.058	+ 6,050	− 351	B_1C_1	−0.058	+ 18,100	− 1,050
A_2B_2	−0.079	+ 4,090	− 323	B_2C_2	−0.079	+ 12,250	− 969
A_3B_3	−0.042	+ 180	− 8	B_3C_3	−0.042	+ 539	− 23
A_4B_4	+0.154	− 1,778	− 274	B_4C_4	+0.154	− 5,310	− 819
A_5B_5	+0.038	+ 180	+ 7	B_5C_5	+0.038	+ 539	+ 20
A_6B_6	+0.001	+ 4,090	+ 4	B_6C_6	+0.001	+ 12,250	+ 12
A_1B_2	+0.056	− 40,100	− 2,245	B_1C_2	−0.065	− 32,800	+ 2,132
A_2B_1	−0.065	+ 47,500	− 3,090	B_2C_1	+0.056	+ 54,900	+ 3,070
A_2B_3	+0.168	− 102,000	− 17,120	B_2C_3	−0.177	− 98,900	+ 17,500
A_3B_2	−0.177	+ 105,000	− 18,600	B_3C_2	+0.168	+ 108,000	+ 18,140
A_3B_4	+0.231	− 107,000	− 24,750	B_3C_4	−0.233	− 108,250	+ 25,220
A_4B_3	−0.233	+ 106,200	− 24,800	B_4C_3	+0.231	+ 105,250	+ 24,350
A_4B_5	+0.033	+ 106,200	+ 3,508	B_4C_5	−0.029	+ 105,250	− 3,060
A_5B_4	−0.029	− 107,000	+ 3,100	B_5C_4	+0.033	− 108,250	− 3,580
A_5B_6	−0.021	+ 105,000	− 2,205	B_5C_6	+0.024	+ 108,000	+ 2,598
A_6B_5	+0.024	− 102,000	− 2,450	B_6C_5	−0.021	− 98,900	+ 2,075
A_6B_1	−0.024	+ 47,500	− 1,140	B_6C_1	+0.020	+ 54,900	+ 1,098
A_1B_6	+0.020	− 40,100	− 802	B_1C_6	−0.024	− 32,800	+ 787
				C_1C_2	−0.003	− 876	+ 3
				C_2C_3	−0.153	+ 22	− 3
				C_3C_4	+0.162	+ 848	+ 137
				C_4C_5	−0.008	+ 848	− 7
				C_5C_6	−0.014	+ 22	− 0
				C_6C_1	+0.017	− 876	− 15

$$\sum_{B}^{A} = -89{,}943$$

$$\sum_{C}^{B} = +87{,}492$$

$$\sum_{C}^{A} = Ex_B = -2{,}451$$

$$\sum_{C}^{B} = +87{,}492$$

Longitudinal 2

The secondary moments are found by applying the auxiliary systems as shown in Figs. 7 and 8, Pt. II.

The tables are similar to those found for longitudinal 3 and are therefore omitted.

The resulting equations are, for the radial system:—

$$4M_{rB2} + M_{rC2} = -63.7\, I_1$$
$$M_{rB2} + 4M_{rC2} = -59.9\, I_1$$

giving

$$M_{rB2} = -13.0\, I_1$$
$$M_{rC2} = -11.7\, I_1$$

and for the tangential system,

$$4M_{tB2} + M_{tC2} = -38.5\, I_1$$
$$M_{tB2} + 4M_{tC2} = -111.0\, I_1$$

giving

$$M_{tB2} = -2.8\, I_1$$
$$M_{tC2} = -27.0\, I_1$$

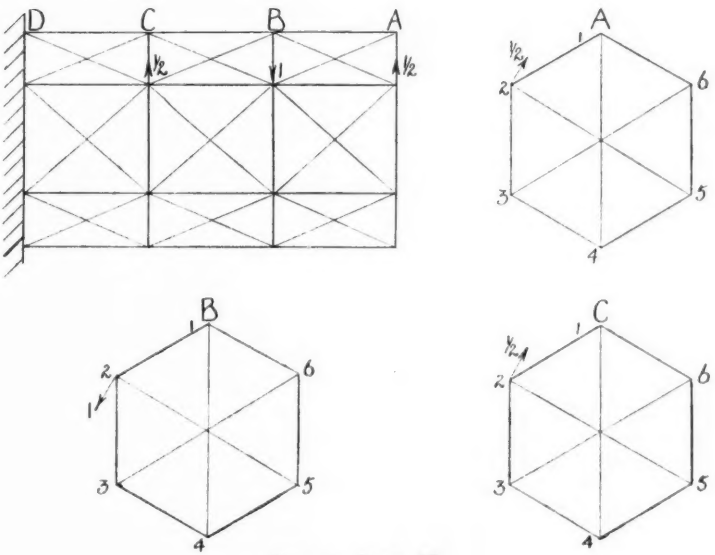


FIG. 8, PART II.

Longitudinal 1

This is a similar case to longitudinal 4. The only secondary moments are due to the radial system (Fig. 9, Pt. II.) and they are found to be

$$M_{rB1} = -24.85\, I_1$$
$$M_{rC1} = -26.78\, I_1$$

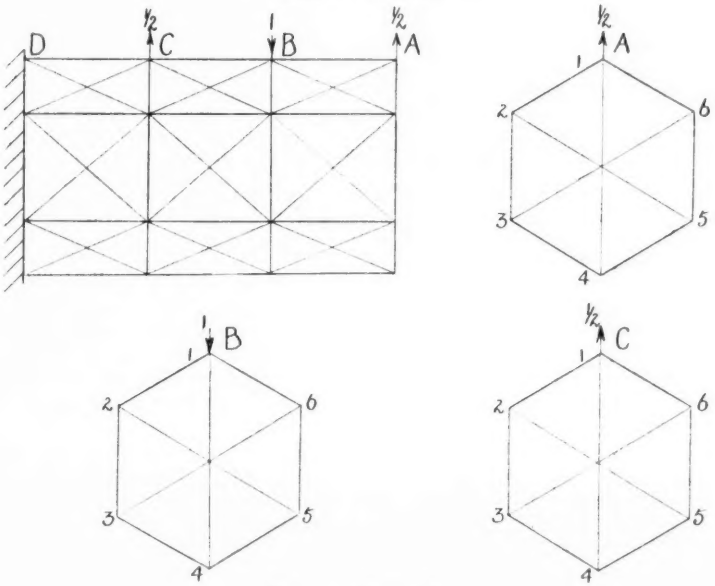


FIG. 9, PART II

All the fixing moments due to the continuity of the longitudinals having been found, the secondary fibre stresses in the longitudinals and the redistribution of shear follow.

In this case the amount of shear taken by the longitudinals acting as beams is very small, the maximum value, which occurs in the bay *CD*, being less than 0.1 per cent.

For the sake of completeness, all the work involved in obtaining the deflections which has not been given above is given in Table 17 to Table 23 at the end of Appendix B.

Conclusions

The complete set of tables needed to obtain the deflections x_B, y_B, etc., have not been given, but it is possible from those above and from Appendix B to estimate the amount of work involved in the application of this method of obtaining a first approximation of the secondary stresses.

Consider a ship with fifteen main frames and fifteen main longitudinals. For each joint in a longitudinal two tables similar to 8 and 9 above will be necessary to provide the equations for the two types of fixing moment at the joint. Neglecting, for the time being, the members in the transverse frames, the number of members included in such a table will be approximately 40 in each bay, since all the diagonal bracing wires will not be operative, or 80 in the whole table. Fifteen of these tables, for one longitudinal, are needed for each type of moment, but the resulting equations are very easily solved.

A total of some 450 tables has therefore to be made for the whole ship. This number may, at first sight, appear prohibitive, but the work involved is not out of proportion to that undertaken in the primary stressing of the ship. The tables of this method, though long, are very simple compared with the complicated work frequently found necessary in the primary stressing of the modern airship.

The diagonal bracing wires of an actual ship will not be initially tensioned and therefore not always operative. This leads to a difference in the methods of determining the loads in the longitudinals and diagonals, but for a fifteen-sided ship the work will not assume undue proportions. Where initial tension does not exist, the diagonal members assumed operative for the auxiliary systems should be those found to be in tension under the external load system.

The transverse members have not been considered above. In a Zeppelin type ship the determination of the loads in them should present no difficulty; a similar method to that of Appendix B could be employed with ease if certain of the bracing members, in the plane of the frame, are assumed inoperative. In ships with transverse frames of more complicated design the determination of the loads in these members, due to the auxiliary systems, may prove more arduous. It is possible, however, to divide the work economically between the primary and secondary stressing. If a representative transverse frame is stressed under unit load at each joint in turn then, by superimposing the resulting loads in the correct proportions, the loads in the members may be obtained for any system, primary or auxiliary, of external load.

In dealing with transverse frames, which are relatively rigid in their own planes and of which the longitudinals form an integral part, it may be found convenient to modify the method by considering the slope at the support, imposed upon the longitudinal by the frame, instead of the actual deflections. It is improbable, however, that the determination of the angle through which the frame twists will be more simple than the determination of the deflections and therefore the method described, which is the more exact, should be employed.

APPENDIX "A"

Extract from

"Influence du Systeme de Triangulation sur les Efforts Secondaires"

BY PROFESSOR Z. BAZANT.*

It is possible to calculate approximately the secondary stresses in a member due to the rigid joints in it, with the assumption that all other members are fixed to these joints by pins, by determining the vertical displacements of the joints in the member and imagining them the movements of the supports of a continuous beam formed by the member under consideration. Then the moments at the supports, that is to say, at the joints in the member are calculated by the Theorem of Three Moments. The equation of three moments in this case follows from the Principle of Virtual Work, which may be written in the form

$$\Sigma (P)\,\Delta p = \int (M)\,M ds/EI$$

in which (P) is an auxiliary force chosen at will.

Δp is the displacement of the point of application in the direction of this force.

(M) is the bending moment arising from the force (P).

M is the bending moment corresponding to the deformation found.

E is the Young's Modulus.

I the moment of inertia of the section of the beam,

and ds an element of the longitudinal axis.

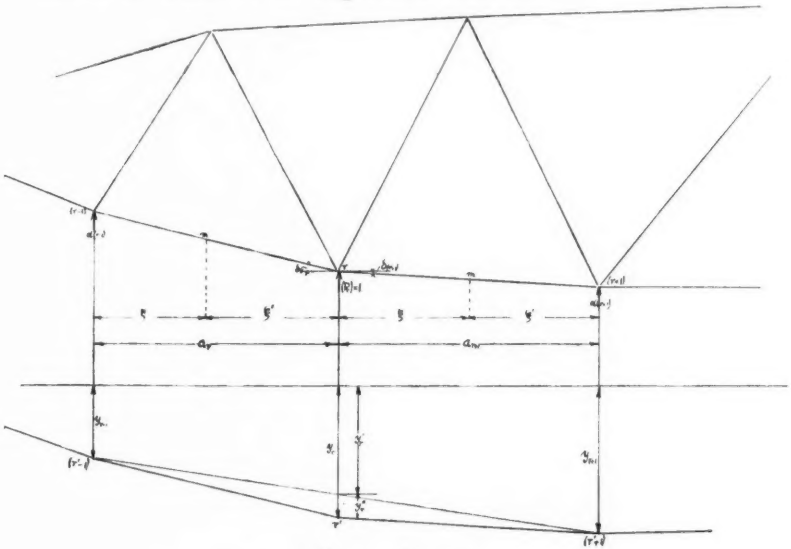


FIG. 1, APP. A.

Applying the Principle of Virtual Work, supposing (Fig. 1, App. A) at the point r there is a vertical auxiliary force $P_r = 1$ and at the adjacent joints reactions a_{r-1}, a_{r+1}, calculated for the part $(r-1)\ (r+1)$ as a simple beam, then

$$a_{r-1} = 1\ (a_{r+1})/(a_r + a_{r+1}) \qquad a_{r+1} = 1\ a_r/(a_r + a_{r-1})$$

* Published by "Masarykova Akademie Práce."

Suppose only the bottom member resists the auxiliary forces a_{r-1}, (P_r), a_{r+1} forming a bent beam.

At some point m of the panel a_r the moment is

$$(M)=a_r \cdot \xi$$

at the point m of the panel a_{r+1} the moment is

$$(M)=a_{r+1} \cdot \xi'$$

ξ (ξ') is the distance of the point m from the extreme left (right) of the panel. The displacements of the joints cause bending moments in the continuous bottom member.

If the moments at the joints $(r-1)$, r, $(r+1)$, are called M_{r-1}, M_r, M_{r+1}, the moments at m of panel a_r, noting that the line representing these moments diagrammatically is straight between joints, is

$$M=(M_{r-1} \cdot \xi' + M_r \cdot \xi)/a_r$$

and by analogy for the point m of panel a_{r+1},

$$M=(M_r\xi' + M_{r+1}\xi)/a_{r+1}$$

Call the vertical displacements of the joints resulting from the deformation of the beam y_{r-1}, y_r and y_{r+1}.

The virtual work of the external auxiliary forces is

$$\Sigma\,(P)\,\Delta p = 1 \cdot y_r - a_{r-1}y_{r-1} - a_{r+1}y_{r+1}$$

Substitute the values of a_{r-1} and a_{r+1},

$$a_{r-1}y_{r-1} + a_{r+1}y_{r+1} = (a_{r+1}y_{r-1} + a_r y_{r+1})/(a_r + a_{r+1}) = y'_r$$

y'_r is the length of the ordinate of joint r cut by the straight line joining joints $(r-1)$, $(r+1)$, then

$$y_r - a_{r-1}y_{r-1} - a_{r+1}y_{r+1} = y''_r$$

y''_r is the deflection of r with respect to $(r-1)$ and $(r+1)$.

Let (M) and M be replaced by their values in the equation of virtual work, assuming generally that in the parts a_r, a_{r+1} of the beam, the moments of inertia are I_r and I_{r+1}; substituting $ds = d\xi/\cos\delta$ gives the equation

$$\Sigma\,(P)\,\Delta p = y''_r = \int_o^{a_r} \{\,(M_{r-1} \cdot \xi' + M_r\xi)/EI_r a_r \cos\delta_r\,\}\; a_{r-1} \cdot \xi \cdot d\xi +$$

$$\int_o^{a_{r+1}} \{\,(M_r \cdot \xi' + M_{r+1} \cdot \xi)/EI_{r+1}a_{r+1}\cos\delta_{r+1}\,\}\; a_{r+1} \cdot \xi' d\xi$$

or

$$y''_r = (a_{r-1}/EI_r a_r \cos\delta_r)\,(M_{r-1}\int_o^{a_r} \xi\xi' d\xi + M_r \int_o^{a_r} \xi^2 d\xi)$$

$$+ (a_{r+1}/EI_{r+1}a_{r+1}\cos\delta_{r+1})\,(M_r\int_o^{a_{r+1}} \xi'^2 d\xi + M_{r+1}\int_o^{a_{r+1}} \xi\xi' d\xi)$$

The integrals have the values

$$\int_o^a \xi^2 d\xi = \int_o^a \xi'^2 d\xi = a^3/3\,; \int_o^a \xi\xi' d\xi = \int_o^a \xi\,(a-\xi)\,d\xi = a^3/6\,;$$

substituting these values and the values of a_{r-1} and a_{r+1} in the preceding equation,

$$\{\,a_{r-1}/(EI_r a_r\,(a_r + a_{r+1})\cos\delta_r)\,\}\;\{\,M_{r-1}\,(a^3{}_r/6) + M_r\,(a^3{}_r/3)\,\}$$
$$+\,\{\,a_r/(EI_{r+1}a_{r+1}\,(a_r + a_{r+1})\cos\delta_{r+1})\,\}\;\{\,M_r\,(a^3{}_{r+1}/3) + M_{r+1}\,(a^3{}_{r+1}/6)\,\} = y''_r$$

or

$$M_{r-1}\,(a_r/\cos\delta_r) + 2M_r\,\{\,(a_r/\cos\delta_r) + (a_{r+1}/\cos\delta_{r+1})\,(I_r/I_{r+1})\,\}$$
$$+ M_{r+1}\,(a_{r+1}/\cos\delta_{r+1})\,(I_r/I_{r+1}) = (6EI_r/a_r a_{r+1})\,(a_r + a_{r+1})\,y''_r \qquad . \qquad (1)$$

This is the equation of three moments and depends only on the influence of the displacement of the support, without taking account of the loads.

This calculation assumes the bottom member continuous and neglects the resistance of the other members, supposing them to be pin-jointed to the bottom member. The end conditions being known, as many equations as there are unknown moments may be written down; in this way the moments at the joints which express the influence of the continuity of the member may be calculated.

For equal panel lengths $a_r = a_{r+1} = a$, equation (1) becomes

$$(M_{r-1}/\cos\delta_r) + 2M_r \{ 1/\cos\delta_r + (1/\cos\delta_{r+1})\ (I_r/I_{r+1}) \} + (M_{r+1}/\cos\delta_{r+1})\ (I_r/I_{r+1}) = 12EI_r/a^2)\ y''_r$$

For a horizontal member $\cos\delta_r = \cos\delta_{r+1} = 1$ and

$$M_{r-1} + 2\ (1 + I_r/I_{r+1})\ M_r + (I_r/I_{r+1})\ M_{r+1} = (12EI_r/a^2)\ y''_r \quad . \quad . \quad (2)$$

Equations (1), (2), show that the moments M at the joints depend on the moments of inertia; the moments M are greater the greater I is, or the greater the resistance of the beam against bending.

The ordinate y''_r which appears in the equation of three moments must be calculated. If the load in a member, due to the auxiliary forces, is called σ_r then

$$\Sigma\ (P)\ \Delta p = \Sigma \sigma_r \Delta s$$

or

$$y''_r = \Sigma \sigma_r \Delta s$$

where Δs is the actual stretch in a member.

APPENDIX "B"

Detailed Calculations of the Loads Due to Primary and Auxiliary Systems

The primary stresses in the members, due to the various load systems applied in Part II., are found by making use of the methods of Appendix 3, R. and M. 800.

The details of the members are

	Cross-Sect. Area (sq. ins.)	Length (inches)	Ω
Longitudinals	0.0668	30.0	75·3
Keel	0.6010	30.0	679.0
Diagonal wires	0.0085	39.06	4·3
Transverses	0.0668	25.0	130.0

The crossed diagonal wires are imagined replaced by an equivalent bar, parallel to the longitudinals and acting through the intersection of the original diagonals. The area of this equivalent bar can be found from the relation

$$\Omega_e = 2\Omega_d\Omega_t/(2\Omega_d + \Omega_t)$$

In this case $\Omega_e = 8.14$ and $S_e = 0.0072$.

Considering the five longitudinals, the keel and the six bars replacing the diagonals, the neutral axis of the cross-section and its moment of inertia about the neutral axis can be found.

The neutral axis is 11.35″ above joint 4 and the moment of inertia about the neutral axis is $287.5^{\text{ins.4}}$.

Load of 100 lbs. Applied at A. (Fig. 2, Pt. 2)

The stresses set up in the members, due to this load, may be taken as resulting from two systems:—

(1) A shear force in direction oy of 100 lbs., which is constant in each bay.

(2) Bending moments about axis ox of amount 1,500, 4,500 and 7,500 lb.-ins. at the centres of the first, second and third bays.

Bending Moments (2) About Axis ox

The loads in the longitudinals and keel and the horizontal component in the equivalent bars, due to a bending moment M, are given by

$$My \times (\text{area of member})/I$$

where y is the distance of the member from the neutral axis and I is the moment of inertia of the cross-section. The loads resulting from the moments (2) are given in Table 10.

TABLE 10.

Member	Bay AB	Bay BC	Bay CD
11	+ 13.45	+ 40.35	+ 67.25
22	+ 9.10	+ 27.30	+ 45.50
33	+ 0.40	+ 1.20	+ 2.00
44	− 35.62	− 106.87	− 178.12
12	+ 0.80	+ 2.40	+ 4.00
23	+ 0.30	+ 0.90	+ 1.50
34	− 0.10	− 0.30	− 0.50

The loads in members 12, 23, 34, are the loads in the diagonal wires, they are obtained as follows:—

Load in equivalent bar replacing wires 34 is

$$(1500 \times 32.40 \times 0.0072)/287.5 = 1.22$$

the actual load in each wire 34

$$= (39.06/30) \times (1.22/2) = 0.8$$

Shear (1) in Direction oy

For the loads resulting from the shear, take the loads in the members of the first bay due to the bending moment 1,500 lb.-ins. The shear must apply horizontal forces to the joints to keep them in equilibrium under these loads. From the bending moment loads, the horizontal force at A_1 is

$$13.45 + 1.22 = 14.67$$

Half the horizontal shear across the panel 12 (F_{12}) is therefore 7.33.

Then the horizontal component in $A_1B_2 = -7.33$

" " " $A_2B_1 = +7.33$

The horizontal force at $A_2 = 9.10 + 0.61 + 0.25 = 9.96$. This equals the resultant of half the horizontal shear in panels 12 and 23, giving

$$F_{23} = 34.58$$

Then the horizontal components in $A_2B_3 = -17.29$

" " " $A_3B_2 = +17.29$

Similarly " " " $A_3B_4 = -17.86$

" " " $A_4B_3 = +17.86$

The actual loads in the wires resulting from the shear (1) are given in Table 11.

TABLE 11.

A_1B_2	− 9·55
A_2B_1	+ 9·55
A_2B_3	− 22·52
A_3B_2	+ 22·52
A_3B_4	− 23·25
A_4B_3	+ 23·25

These loads are the same in each bay and therefore the final results in Table 6, column 3, due to 100 lbs. applied at A_4 are found by adding Table 11 to each column of Table 10 in turn.

Transverse Members

The loads in these members are found by considering the transverse frame as a separate structure, loaded in its own plane by the external loads and the diagonal wires. The loading on the frame will then be as in Fig. 1, App. B.

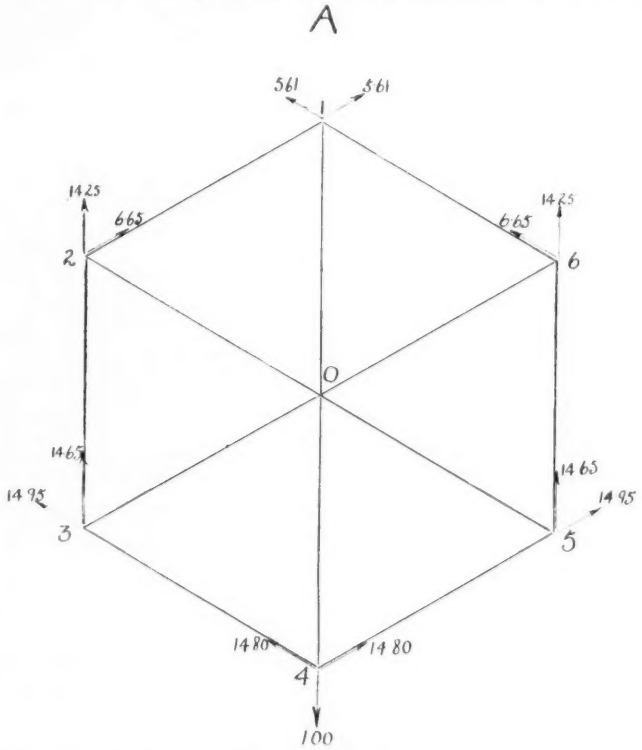


FIG. 1, APP. B.

If the tensions in the members 43, 32, 21, 04, are assumed to be T_1, T_2, T_3 and R, then

$$T_1 = 85.2 - R$$
$$T_2 = 55.6 - R$$
$$T_3 = 34.7 - R$$

From considerations of strain energy, neglecting the energy in the radial bracing, which is composed of members of large cross-section compared with the transverses,

$$R_1 = +58.5$$

Therefore

$$A_4A_3 = +26.70$$
$$A_3A_2 = -\ 2.90$$
$$A_2A_1 = -23.80$$

The loads in the transverse frames B and C are found in the same way.

Auxiliary Systems

The effect of the auxiliary systems is found most simply by stressing the first bay of the structure under the five following systems applied to the bulkhead A, Fig. 2, Pt. II. :—

(1) A shear force of 1 lb. in the direction *yo*.
(2) A bending moment of 15 lb.-ins. about the axis *ox* at the centre of the bay.
(3) A shear force of 1 lb. in the direction *ox*.
(4) A bending moment of 15 lb.-ins. about the axis *oy* at the centre of the bay.
(5) A clockwise twisting couple of 12.5 lb.-ins.

Shear Force (1) and Bending Moment (2)

The loads from (1) and (2) are given in Tables 13 and 12. They are merely the loads of Tables 11 and 10 divided by 100 and of opposite sign.

TABLE 12.

A_1B_1	−0.1345	A_1B_2	−0.0080
A_2B_2	−0.0910	A_2B_1	−0.0080
A_3B_3	−0.0040	A_2B_3	−0.0030
A_4B_4	+0.3562	A_3B_2	−0.0030
		A_3B_4	+0.0010
		A_4B_3	+0.0010

TABLE 13.

A_1B_2	+0.0955
A_2B_1	−0.0955
A_2B_3	+0.2252
A_3B_2	−0.2252
A_3B_4	+0.2325
A_4B_3	−0.2325

Bending Moment (4)

The bending moment (4) may be treated as was (2) above. The moment of inertia of the cross-section about the neutral axis *oy* is $135.5^{\text{ins.4}}$ and the resulting loads are given in Table 14.

TABLE 14.

A_1B_1	0	A_1B_2	+0.005625	A_4B_5	−0.005625
A_2B_2	+0.16	A_2B_1	+0.005625	A_5B_4	−0.005625
A_3B_3	+0.16	A_2B_3	+0.011250	A_5B_6	−0.011250
A_4B_4	0	A_3B_2	+0.011250	A_6B_5	−0.011250
A_5B_5	−0.16	A_3B_4	+0.005625	A_6B_1	−0.005625
A_6B_6	−0.16	A_4B_3	+0.005625	A_1B_6	−0.005625

Shear Force (3)

Let the horizontal actions at the joints due to the bending loads from (4) be Z_1, Z_2, Z_3, Z_4, then as above,

$$\tfrac{1}{2}(F_{23}-F_{12})=Z_2$$
$$\tfrac{1}{2}(F_{34}-F_{23})=Z_3$$

Therefore

$$F_{23}=F_{12}+0.34595$$
$$F_{34}=F_{12}+0.69190$$
$$\Sigma F=0$$

and so

$$F_{12}=-0.34595$$
$$F_{23}=0$$
$$F_{34}=+0.34595$$

The horizontal components of loads in the wires are therefore ± 0.172975 and the loads in the wires, due to the shear (3), are given in Table 15.

TABLE 15.

A_1B_2	$+0.225$	A_4B_5	-0.225
A_2B_1	-0.225	A_5B_4	$+0.225$
A_2B_3	0	A_5B_6	0
A_3B_2	0	A_6B_5	0
A_3B_4	-0.225	A_6B_1	$+0.225$
A_4B_3	$+0.225$	A_1B_6	-0.225

Twisting Couple (5)

Due to a twisting couple T, the horizontal shear force (F) across each panel is given by

$$F=T\times l/2 \text{ (area of cross-section)}$$

where l is the length of bay.

In this case $F=0.1154$.

The horizontal component of load in the wires is ± 0.0577 and the actual loads are given in Table 16.

The longitudinals are unstressed.

TABLE 16.

A_1B_2	$+0.075$	A_4B_5	$+0.075$
A_2B_1	-0.075	A_5B_4	-0.075
A_2B_3	$+0.075$	A_5B_6	$+0.075$
A_3B_2	-0.075	A_6B_5	-0.075
A_3B_4	$+0.075$	A_6B_1	$+0.075$
A_4B_3	-0.075	A_1B_6	-0.075

With the help of these tables the loads in the longitudinals and diagonal wires, due to the six auxiliary systems, may be at once obtained as follows:—

AUXILIARY SYSTEM 1 (Fig. 3, Pt. II.).

Bay AB.—$\frac{1}{2}\times$ Table 12 $+\frac{1}{2}\times$ Table 13.
Bay BC.—$\frac{1}{2}\times$ Table 12 $-\frac{1}{2}\times$ Table 13.

These loads are given in Table 7, column 1.

AUXILIARY SYSTEM 2 (Fig. 5, Pt. II.).

Bay AB.—$\frac{1}{2}\cos 60°$ [Table 12 + Table 13] + $\frac{1}{2}\sin 60°$ [Table 14 + Table 15].

Bay BC.—$\frac{1}{2}\cos 60°$ [Table 12 − Table 13] + $\frac{1}{2}\sin 60°$ [Table 14 − Table 15].

AUXILIARY SYSTEM 3 (Fig. 6, Pt. II.).

Bay AB.—$\frac{1}{2}\sin 60°$ [Table 12 + Table 13] − $\frac{1}{2}\cos 60°$ [Table 14 + Table 15] + Table 16.

Bay BC.—$\frac{1}{2}\sin 60°$ [Table 12 − Table 13] − $\frac{1}{2}\cos 60°$ [Table 14 − Table 15] − Table 16.

These loads are given in Table 9, column 1.

AUXILIARY SYSTEM 4 (Fig. 7, Pt. II.).

Bay AB.—$\frac{1}{2}\cos 60°$ [Table 12 + Table 13] − $\frac{1}{2}\sin 60°$ [Table 14 + Table 15].

Bay BC.—$\frac{1}{2}\cos 60°$ [Table 12 − Table 13] − $\frac{1}{2}\sin 60°$ [Table 14 − Table 15].

AUXILIARY SYSTEM 5 (Fig. 8, Pt. II.).

Bay AB.—$\frac{1}{2}\sin 60°$ [Table 12 + Table 13] + $\frac{1}{2}\cos 60°$ [Table 14 + Table 15] + Table 16.

Bay BC.—$\frac{1}{2}\sin 60°$ [Table 12 − Table 13] + $\frac{1}{2}\cos 60°$ [Table 14 − Table 15] − Table 16.

AUXILIARY SYSTEM 6 (Fig. 9, Pt. II.).

Bay AB.—$\frac{1}{2}\times$ Table 12 + $\frac{1}{2}\times$ Table 13.

Bay BC.—$\frac{1}{2}\times$ Table 12 − $\frac{1}{2}\times$ Table 13.

The loads in the transverse members may be found from the above by the methods already employed in this appendix.

When the external auxiliary loads are applied at bulkheads B, C, D, instead of at A, B, C, as assumed above, then the loads in the members of bays BC and CD are the same as those found in the corresponding members of bays AB and BC respectively.

The tables of loads, due to the auxiliary systems, which have not been given in Part II. will be found at the end of this appendix in Tables 17 to 23.

TABLE 17.

LONGITUDINAL 3.

RADIAL AUXILIARY SYSTEM AT *ABC*.

Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$	Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$
A_1A_2	+0.008	− 8,910	− 72	B_1B_2	+0.009	− 438	− 4
A_2A_3	−0.123	− 1,086	+ 133	B_2B_3	+0.261	+ 11	+ 3
A_3A_4	−0.133	+ 10,000	− 1,330	B_3B_4	+0.240	+ 424	+ 102
A_4A_5	−0.005	+ 10,000	− 50	B_4B_5	−0.004	+ 424	− 2
A_5A_6	+0.133	− 1,086	− 144	B_5B_6	−0.241	+ 11	− 3
A_6A_1	+0.121	− 8,910	− 1,080	B_6B_1	−0.265	− 438	+ 116
A_1B_1	−0.034	+ 6,050	− 206	B_1C_1	−0.034	+ 18,100	− 615
A_2B_2	+0.047	+ 4,090	+ 192	B_2C_2	+0.047	+ 12,250	+ 576
A_3B_3	+0.068	+ 180	+ 12	B_3C_3	+0.068	+ 539	+ 36
A_4B_4	+0.089	− 1,778	− 158	B_4C_4	+0.089	− 5,310	− 473
A_5B_5	−0.070	+ 180	− 12	B_5C_5	−0.070	+ 539	− 38
A_6B_6	−0.092	+ 4,090	− 376	B_6C_6	−0.092	+ 12,250	− 1,128
A_1B_2	+0.122	− 40,100	− 4,900	B_1C_2	−0.121	− 32,800	+ 3,970
A_2B_1	−0.121	+ 47,500	− 5,750	B_2C_1	+0.122	+ 54,900	+ 6,700
A_2B_3	+0.060	−102,000	− 6,120	B_2C_3	−0.052	− 98,900	+ 5,140
A_3B_2	−0.052	+105,000	− 5,460	B_3C_2	+0.060	+108,000	+ 6,480
A_3B_4	−0.037	−107,000	+ 3,960	B_3C_4	+0.042	−108,250	− 4,550
A_4B_3	+0.042	+106,200	+ 4,460	B_4C_3	−0.037	+105,250	− 3,900
A_4B_5	−0.158	+106,200	−16,800	B_4C_5	+0.153	+105,250	+16,120
A_5B_4	+0.153	−107,000	−16,390	B_5C_4	−0.158	−108,250	+17,120
A_5B_6	−0.062	+105,000	− 6,510	B_5C_6	+0.051	+108,000	+ 5,500
A_6B_5	+0.051	−102,000	− 5,200	B_6C_5	−0.062	− 98,900	+ 6,130
A_6B_1	+0.069	+ 47,500	+ 3,280	B_6C_1	−0.078	+ 54,900	− 4,280
A_1B_6	−0.078	− 40,100	+ 3,130	B_1C_6	+0.069	− 32,800	− 2,260
				C_1C_2	+0.008	− 876	− 7
				C_2C_3	−0.123	+ 22	− 3
				C_3C_4	−0.133	+ 848	− 113
				C_4C_5	−0.005	+ 848	− 4
				C_5C_6	+0.133	+ 22	+ 3
				C_6C_1	+0.121	− 876	− 106

$$\sum_{A}^{B} = -55{,}391$$

$$\sum_{B}^{C} = +50{,}510$$

$$\sum_{A}^{C} = Ey_B = -4{,}881$$

$$\sum_{C}^{B} = +50{,}510$$

TABLE 18.

LONGITUDINAL 3.

RADIAL AUXILIARY SYSTEM AT *BCD*.

Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$	Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$
B_1B_2	+0.008	− 438	− 3	C_1C_2	+0.009	− 876	− 8
B_2B_3	−0.123	+ 11	− 1	C_2C_3	+0.261	+ 22	+ 5
B_3B_4	−0.133	+ 424	− 56	C_3C_4	+0.240	+ 848	+ 204
B_4B_5	−0.005	+ 424	− 2	C_4C_5	−0.004	+ 848	− 3
B_5B_6	+0.133	+ 11	+ 1	C_5C_6	−0.241	+ 22	− 5
B_6B_1	+0.121	− 438	− 53	C_6C_1	−0.265	− 876	+ 232
B_1C_1	−0.034	+ 18,100	− 615	C_1D_1	−0.034	+ 30,200	− 1,028
B_2C_2	+0.047	+ 12,250	+ 576	C_2D_2	+0.047	+ 20,420	+ 964
B_3C_3	+0.068	+ 539	+ 37	C_3D_3	+0.068	+ 900	+ 61
B_4C_4	+0.089	− 5,310	− 473	C_4D_4	+0.089	− 8,890	− 790
B_5C_5	−0.070	+ 539	− 38	C_5D_5	−0.070	+ 900	− 63
B_6C_6	−0.092	+ 12,250	− 1,128	C_6D_6	−0.092	+ 20,420	− 1,880
B_1C_2	+0.122	− 32,800	− 4,000	C_1D_2	−0.121	− 25,450	+ 3,080
B_2C_1	−0.121	+ 54,900	− 6,640	C_2D_1	+0.122	+ 62,200	+ 7,600
B_2C_3	+0.060	− 98,900	− 5,934	C_2D_3	−0.052	− 96,000	+ 5,000
B_3C_2	−0.052	+ 108,000	− 5,610	C_3D_2	+0.060	+ 110,800	+ 6,648
B_3C_4	−0.037	− 108,250	+ 4,010	C_3D_4	+0.042	− 109,000	− 4,580
B_4C_3	+0.042	+ 105,250	+ 4,420	C_4D_3	−0.037	+ 104,250	− 3,860
B_4C_5	−0.158	+ 105,250	− 16,640	C_4D_5	+0.153	+ 104,250	+ 15,960
B_5C_4	+0.153	− 108,250	− 16,600	C_5D_4	−0.158	− 109,000	+ 17,200
B_5C_6	−0.062	+ 108,000	− 6,700	C_5D_6	+0.051	+ 110,800	+ 5,650
B_6C_5	+0.051	− 98,900	− 5,050	C_6D_5	−0.062	− 96,000	+ 5,950
B_6C_1	+0.069	+ 54,900	+ 3,780	C_6D_1	−0.078	+ 62,200	− 4,850
B_1C_6	−0.078	− 32,800	+ 2,560	C_1D_6	+0.069	− 25,450	− 1,758

$$\sum_{C}^{B} = -54{,}159$$

$$\sum_{D}^{C} = +49{,}729$$

$$\sum_{D}^{B} = Ey_c = -4{,}430$$

$$\sum_{D}^{C} = +49{,}729$$

TABLE 19.

LONGITUDINAL 3.

TANGENTIAL AUXILIARY SYSTEM AT *BCD*.

Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$	Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$
B_1B_2	−0.003	− 438	+ 1	C_1C_2	+0.020	− 876	− 17
B_2B_3	−0.153	+ 11	− 2	C_2C_3	+0.314	+ 22	+ 7
B_3B_4	+0.162	+ 424	+ 68	C_3C_4	−0.326	+ 848	− 276
B_4B_5	−0.008	+ 424	− 3	C_4C_5	+0.008	+ 848	+ 7
B_5B_6	−0.014	+ 11	− 0	C_5C_6	+0.020	+ 22	+ 0
B_6B_1	+0.017	− 438	− 7	C_6C_1	−0.038	− 876	+ 33
B_1C_1	−0.058	+ 18,100	− 1,050	C_1D_1	−0.058	+ 30,200	− 1,750
B_2C_2	−0.079	+ 12,250	− 969	C_2D_2	−0.079	+ 20,420	− 1,613
B_3C_3	−0.042	+ 539	− 23	C_3D_3	−0.042	+ 900	− 38
B_4C_4	+0.154	− 5,310	− 819	C_4D_4	+0.154	− 8,800	− 1,368
B_5C_5	+0.038	+ 539	+ 20	C_5D_5	+0.038	+ 900	+ 34
B_6C_6	+0.001	+ 12,250	+ 12	C_6D_6	+0.001	+ 20,420	+ 20
B_1C_2	+0.056	− 32,800	− 1,840	C_1D_2	−0.065	− 25,450	+ 1,658
B_2C_1	−0.065	+ 54,900	− 3,570	C_2D_1	+0.056	+ 62,200	+ 3,481
B_2C_3	+0.168	− 98,900	− 16,600	C_2D_3	−0.177	− 96,000	+ 17,000
B_3C_2	−0.177	+ 108,000	− 19,110	C_3D_2	+0.168	+ 110,800	+ 18,600
B_3C_4	+0.231	− 108,250	− 25,000	C_3D_4	−0.233	− 109,000	+ 25,400
B_4C_3	−0.233	+ 105,250	− 24,580	C_4D_3	+0.231	+ 104,250	+ 24,050
B_4C_5	+0.033	+ 105,250	+ 3,480	C_4D_5	−0.029	+ 104,250	− 3,025
B_5C_4	−0.029	− 108,250	+ 3,140	C_5D_4	+0.033	− 109,000	− 3,600
B_5C_6	−0.021	+ 108,000	− 2,265	C_5D_6	+0.024	+ 110,800	+ 2,660
B_6C_5	+0.024	− 98,900	− 2,372	C_6D_5	−0.021	− 96,000	+ 2,018
B_6C_1	−0.024	+ 54,900	− 1,320	C_6D_1	+0.020	+ 62,200	+ 1,244
B_1C_6	+0.020	− 32,800	− 656	C_1D_6	−0.024	− 25,450	+ 612

$$\sum_{C}^{B} = -93{,}465$$

$$\sum_{D}^{C} = +85{,}137$$

$$\sum_{D}^{B} = Ex_C = -8{,}328$$

$$\sum_{D}^{C} = +85{,}137$$

TABLE 20.

LONGITUDINAL 2.

RADIAL AUXILIARY SYSTEM AT *ABC*.

Member	σ_1		$E\cdot\Delta l$		$\sigma_1\cdot E\cdot\Delta l$
A_1A_2	+0.124	−	8,910	−	1,105
A_2A_3	+0.136	−	1,086	−	147
A_3A_4	−0.002	+	10,000	−	20
A_4A_5	−0.130	+	10,000	−	1,300
A_5A_6	−0.121	−	1,086	+	131
A_6A_1	−0.006	−	8,910	+	53
A_1B_1	−0.034	+	6,050	−	206
A_2B_2	−0.092	+	4,090	−	376
A_3B_3	−0.070	+	180	−	13
A_4B_4	+0.089	−	1,778	−	158
A_5B_5	+0.068	+	180	+	12
A_6B_6	+0.047	+	4,090	+	192
A_1B_2	−0.078	−	40,100	+	3,122
A_2B_1	+0.069	+	47,500	+	3,280
A_2B_3	+0.051	−	102,000	−	5,205
A_3B_2	−0.062	+	105,000	−	6,510
A_3B_4	+0.153	−	107,000	−	16,390
A_4B_3	−0.158	+	106,200	−	16,800
A_4B_5	+0.042	+	106,200	+	4,460
A_5B_4	−0.037	−	107,000	+	3,960
A_5B_6	−0.052	+	105,000	−	5,460
A_6B_5	+0.060	−	102,000	−	6,120
A_6B_1	−0.121	+	47,500	−	5,750
A_1B_6	+0.121	−	40,100	−	4,860

$$\sum_{B}^{A} = -55{,}210$$

$$\sum_{C}^{B} = +50{,}432$$

$$\sum_{C}^{A} = Ey_B = -4{,}778$$

Member	σ_1		$E\cdot\Delta l$		$\sigma_1\cdot E\cdot\Delta l$
B_1B_2	−0.239	−	438	+	105
B_2B_3	−0.259	+	11	−	3
B_3B_4	+0.007	+	424	+	3
B_4B_5	+0.251	+	424	+	106
B_5B_6	+0.229	+	11	+	3
B_6B_1	+0.009	−	438	−	4
B_1C_1	−0.034	+	18,100	−	615
B_2C_2	−0.092	+	12,250	−	1,128
B_3C_3	−0.070	+	539	−	38
B_4C_4	+0.089	−	5,310	−	474
B_5C_5	+0.068	+	539	+	37
B_6C_6	+0.047	+	12,250	+	576
B_1C_2	+0.069	−	32,800	−	2,260
B_2C_1	−0.078	+	54,900	−	4,280
B_2C_3	−0.062	−	98,900	+	6,120
B_3C_2	+0.051	+	108,000	+	5,510
B_3C_4	−0.158	−	108,250	+	17,100
B_4C_3	+0.153	+	105,250	+	16,110
B_4C_5	−0.037	+	105,250	−	3,900
B_5C_4	+0.042	−	108,250	−	4,550
B_5C_6	+0.060	+	108,000	+	6,480
B_6C_5	−0.052	−	98,900	+	5,140
B_6C_1	+0.121	+	54,900	+	6,640
B_1C_6	−0.121	−	32,800	+	3,970
C_1C_2	+0.124	−	876	−	109
C_2C_3	+0.136	+	22	+	3
C_3C_4	−0.002	+	848	−	2
C_4C_5	−0.130	+	848	−	110
C_5C_6	−0.121	+	22	−	3
C_6C_1	−0.006	−	876	+	5

$$\sum_{C}^{B} = +50{,}432$$

TABLE 21.

LONGITUDINAL 2.

RADIAL AUXILIARY SYSTEM AT *BCD*.

Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$	Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$
B_1B_2	+0.124	− 438	− 54	C_1C_2	−0.239	− 876	+ 210
B_2B_3	+0.136	+ 11	+ 1	C_2C_3	−0.259	+ 22	− 6
B_3B_4	−0.002	+ 424	− 1	C_3C_4	+0.007	+ 848	+ 6
B_4B_5	−0.130	+ 424	− 55	C_4C_5	+0.251	+ 848	+ 212
B_5B_6	−0.121	+ 11	− 1	C_5C_6	+0.229	+ 22	+ 5
B_6B_1	−0.006	− 438	+ 3	C_6C_1	+0.009	− 876	− 8
B_1C_1	−0.034	+ 18,100	− 615	C_1D_1	−0.034	+ 30,200	− 1,055
B_2C_2	−0.092	+ 12,250	− 1,128	C_2D_2	−0.092	+ 20,420	− 1,881
B_3C_3	−0.070	+ 539	− 38	C_3D_3	−0.070	+ 900	− 63
B_4C_4	+0.089	− 5,310	− 474	C_4D_4	+0.089	− 8,890	− 790
B_5C_5	+0.068	+ 539	+ 37	C_5D_5	+0.068	+ 900	+ 61
B_6C_6	+0.047	+ 12,250	+ 576	C_6D_6	+0.047	+ 20,420	+ 961
B_1C_2	−0.078	− 32,800	+ 2,560	C_1D_2	+0.069	− 25,450	− 1,755
B_2C_1	+0.069	+ 54,900	+ 3,780	C_2D_1	−0.078	+ 62,200	− 4,850
B_2C_3	+0.051	− 98,900	− 5,040	C_2D_3	−0.062	− 96,000	+ 5,950
B_3C_2	−0.062	+ 108,000	− 6,700	C_3D_2	+0.051	+ 110,800	+ 5,650
B_3C_4	+0.153	− 108,250	− 16,590	C_3D_4	−0.158	− 109,000	+ 17,200
B_4C_3	−0.158	+ 105,250	− 16,640	C_4D_3	+0.153	+ 104,250	+ 15,950
B_4C_5	+0.042	+ 105,250	+ 4,420	C_4D_5	−0.037	+ 104,250	− 3,860
B_5C_4	−0.037	− 108,250	+ 4,010	C_5D_4	+0.042	− 109,000	− 4,580
B_5C_6	−0.052	+ 108,000	− 5,610	C_5D_6	+0.060	+ 110,800	+ 6,648
B_6C_5	+0.060	− 98,900	− 5,934	C_6D_5	−0.052	− 96,000	+ 4,990
B_6C_1	−0.121	+ 54,900	− 6,640	C_6D_1	+0.121	+ 62,200	+ 7,530
B_1C_6	+0.121	− 32,800	− 3,970	C_1D_6	−0.121	− 25,450	+ 3,080
		$\sum_{C}^{B} =$	− 54,103			$\sum_{D}^{C} =$	+ 49,605
		$\sum_{D}^{C} =$	+ 49,605				
		$\sum_{D}^{B} = Ey_c =$	− 4,498				

TABLE 22.

LONGITUDINAL 2.

TANGENTIAL AUXILIARY SYSTEM AT *ABC*.

Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$	Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$
A_1A_2	−0.175	− 8,910	+ 1,560	B_1B_2	+0.351	− 438	− 154
A_2A_3	+0.178	− 1,086	− 193	B_2B_3	−0.364	+ 11	− 4
A_3A_4	−0.008	+ 10,000	− 80	B_3B_4	+0.008	+ 424	+ 3
A_4A_5	−0.031	+ 10,000	− 310	B_4B_5	+0.058	+ 424	+ 25
A_5A_6	+0.038	− 1,086	− 41	B_5B_6	−0.068	+ 11	− 1
A_6A_1	−0.006	− 8,910	+ 53	B_6B_1	+0.020	− 438	− 9
A_1B_1	−0.058	+ 6,050	− 351	B_1C_1	−0.058	+ 18,100	− 1,050
A_2B_2	+0.001	+ 4,090	+ 4	B_2C_2	+0.001	+ 12,250	+ 12
A_3B_3	+0.038	+ 180	+ 7	B_3C_3	+0.038	+ 539	+ 20
A_4B_4	+0.154	− 1,778	− 274	B_4C_4	+0.154	− 5,310	− 820
A_5B_5	−0.042	+ 180	− 8	B_5C_5	−0.042	+ 539	− 23
A_6B_6	−0.079	+ 4,090	− 323	B_6C_6	−0.079	+ 12,250	− 965
A_1B_2	+0.171	− 40,100	− 6,850	B_1C_2	−0.175	− 32,800	+ 5,740
A_2B_1	−0.175	+ 47,500	− 8,310	B_2C_1	+0.171	+ 54,900	+ 9,390
A_2B_3	+0.174	− 102,000	− 17,740	B_2C_3	−0.171	− 98,900	+ 16,900
A_3B_2	−0.171	+ 105,000	− 17,950	B_3C_2	+0.174	+ 108,000	+ 18,800
A_3B_4	+0.121	− 107,000	− 12,930	B_3C_4	−0.118	− 108,250	+ 12,800
A_4B_3	−0.118	+ 106,200	− 12,550	B_4C_3	+0.121	+ 105,250	+ 12,730
A_4B_5	−0.082	+ 106,200	− 8,710	B_4C_5	+0.081	+ 105,250	+ 8,540
A_5B_4	+0.081	− 107,000	− 8,660	B_5C_4	−0.082	− 108,250	+ 8,890
A_5B_6	−0.027	+ 105,000	− 2,840	B_5C_6	+0.018	+ 108,000	+ 1,942
A_6B_5	+0.018	− 102,000	− 1,838	B_6C_5	−0.027	− 98,900	+ 2,665
A_6B_1	+0.086	+ 47,500	+ 4,090	B_6C_1	−0.095	+ 54,900	− 5,210
A_1B_6	−0.095	− 40,100	+ 3,820	B_1C_6	+0.086	− 32,800	− 2,820
				C_1C_2	−0.175	− 876	+ 153
				C_2C_3	+0.178	+ 22	+ 4
				C_3C_4	−0.008	+ 848	− 7
				C_4C_5	−0.031	+ 848	− 26
				C_5C_6	+0.038	+ 22	+ 1
				C_6C_1	−0.006	− 876	+ 5

$$\sum_{B}^{A} = -90,424$$

$$\sum_{C}^{B} = +87,531$$

$$\sum_{C}^{A} = Ex_B = -2,893$$

$$\sum_{C}^{B} = +87,531$$

TABLE 23.

LONGITUDINAL 2.

TANGENTIAL AUXILIARY SYSTEM AT *BCD*.

Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$	Member	σ_1	$E\cdot\Delta l$	$\sigma_1\cdot E\cdot\Delta l$
B_1B_2	−0.175	− 438	+ 76	C_1C_2	+0.351	− 876	− 308
B_2B_3	+0.178	+ 11	+ 2	C_2C_3	−0.364	+ 22	− 8
B_3B_4	−0.008	+ 424	− 3	C_3C_4	+0.008	+ 848	+ 7
B_4B_5	−0.031	+ 424	− 13	C_4C_5	+0.058	+ 848	+ 49
B_5B_6	+0.038	+ 11	+ 0	C_5C_6	−0.068	+ 22	− 1
B_6B_1	−0.006	− 438	+ 3	C_6C_1	+0.020	− 876	− 18
B_1C_1	−0.058	+ 18,100	− 1,050	C_1D_1	−0.058	+ 30,200	− 1,750
B_2C_2	+0.001	+ 12,250	+ 12	C_2D_2	+0.001	+ 20,420	+ 20
B_3C_3	+0.038	+ 539	+ 20	C_3D_3	+0.038	+ 900	+ 34
B_4C_4	+0.154	− 5,310	− 820	C_4D_4	+0.154	− 8,890	− 1,368
B_5C_5	−0.042	+ 539	− 23	C_5D_5	−0.042	+ 900	− 37
B_6C_6	−0.079	+ 12,250	− 965	C_6D_6	−0.079	+ 20,420	− 1,614
B_1C_2	+0.171	− 32,800	− 5,610	C_1D_2	−0.175	− 25,450	+ 4,460
B_2C_1	−0.175	+ 54,900	− 9,600	C_2D_1	+0.171	+ 62,200	+ 10,620
B_2C_3	+0.174	− 98,900	− 17,200	C_2D_3	−0.171	− 96,000	+ 16,410
B_3C_2	−0.171	+ 108,000	− 18,480	C_3D_2	+0.174	+ 110,800	+ 19,250
B_3C_4	+0.121	− 108,250	− 13,100	C_3D_4	−0.118	− 109,000	+ 12,880
B_4C_3	−0.118	+ 105,250	− 12,420	C_4D_3	+0.121	+ 104,250	+ 12,620
B_4C_5	−0.082	+ 105,250	− 8,650	C_4D_5	+0.081	+ 104,250	+ 8,450
B_5C_4	+0.081	− 108,250	− 8,770	C_5D_4	−0.082	− 109,000	+ 8,950
B_5C_6	−0.027	+ 108,000	− 2,920	C_5D_6	+0.018	+ 110,800	+ 1,992
B_6C_5	+0.018	− 98,900	− 1,780	C_6D_5	−0.027	− 96,000	+ 2,590
B_6C_1	+0.086	+ 54,900	+ 4,710	C_6D_1	−0.095	+ 62,200	− 5,910
B_1C_6	−0.095	− 32,800	+ 3,120	C_1D_6	+0.086	− 25,450	− 2,190

$$\sum_{C}^{B} = -93{,}461$$

$$\sum_{D}^{C} = +85{,}128$$

$$\sum_{D}^{B} = Ex_0 = -8{,}333$$

$$\sum_{D}^{C} = +85{,}128$$

RESEARCH ON CHANNEL WALL INTERFERENCE*

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Summary

An investigation of channel wall interference with the object of verifying experimentally the accuracy of the results given by the theory developed by Prandtl and extended by Glauert. Tests were made, with cylinders and aerofoils, using the "mirror" method, for some ten different channel sizes. The results show:

1. The presence of the channel walls results in an increase in the drag of a symmetrical body, such as a cylinder.
2. Agreement with the theoretical corrections for incidence and drag for channels of reasonable size.
3. The extent and nature of the wall interference effects on the aerodynamic characteristics of aerofoils in much restricted streams, beyond the limits of applicability of the theory.

General Introduction

The wind channel is one of the principal sources of numerical information for the development of aerodynamics and the design of aircraft. In the wind channel a reduced scale model of the complete aircraft or aircraft component is supported in a moving current of air, and the forces and moments exerted upon it are measured on an aerodynamic balance. The conditions of the wind channel test differ materially from those of the full-sized aircraft in free flight, and hence the results obtained from the model test are subject to various corrections before becoming strictly applicable to full scale free flight conditions.

The modifying influences which must be taken into consideration when using wind channel† data may be listed as follows:—

1. Those due to the model.
   (*a*) Size.
   (*b*) Nature of surface.
2. Those due to support of model.
3. Those due to the wind channel.
   (*a*) Type (open or closed).
   (*b*) Cross section. I. Shape. II. Size.
4. Those due to the air current.
   (*a*) Speed.
   (*b*) Turbulence or flow texture.

Some of these, notably those due to model, model support and air speed, have been much investigated, and considerable experimental information accumulated. On others, but little experimental work has as yet been done, and knowledge is limited. This particular research was concerned with the corrections necessary due to the wind channel, on which experimental information is lacking.

The work was performed in the Aerodynamic Laboratory of the University of Toronto, under the auspices of the School of Engineering Research. A grant

* Aeronautical Research Paper, No. 17 (Canada).
† Atmospheric pressure channels only are here considered.

was made by the Honorary Advisory Council for Industrial and Scientific Research for the purchase of the models and other special apparatus, including the erection of the cellular wall across the building, used in the research.

After the design and construction of the special holding devices and other necessary apparatus during the summer of 1924, preliminary tests were made on cylinders during August and September, 1924. The bulk of the work on cylinders, and all of that on aerofoils, was done from May to September, 1925. Pressure of other work prevented the preparation of the results until the present—August, 1926.

Aeronautical Research Paper No 17. **TABLE Nº 1.**
UNIVERSITY OF TORONTO
CHANNEL WALL INTERFERENCE Aug 24.1926
CYLINDER DRAG COEFFICIENTS

Cylinder 1"x18" Channel Width-4.0ft.

Channel Height-ft	Air Speed 20	30	40	50
4.000	0.464	0.473	0.477	0.483
1.583	0.461	0.471	0.484	0.487
1.056	0.463	0.471	0.484	0.494
0.637	0.476	0.488	0.491	0.504
0.488	0.480	0.482	0.494	0.497
0.352	0.500	0.524	0.518	0.534
0.284	0.521	0.527	0.548	0.549
0.249	0.539	0.564	0.570	0.578
0.227	0.544	0.556	0.566	0.576
0.208	0.573	0.586	0.575	0.581
0.205	0.592	0.589	0.580	0.597
0.204	0.566	0.574	0.577	0.592
0.195	0.546	0.552	0.591	0.598
0.192	0.450	0.500	0.527	0.580
0.179	0.447	0.450	0.453	0.460
0.174	0.430	0.413	0.419	0.443
0.161	0.444	0.426	0.429	0.442
0.157	0.452	0.438	0.430	0.441
0.150	0.454	0.440	0.442	0.446

Cylinder 2"x18" Channel Width-4.0ft.

Channel Height-ft	Air Speed 20	30	40	50
4.000	0.424	0.422	0.430	0.432
1.500	0.441	0.450	0.453	0.455
1.166	0.445	0.447	0.453	0.456
0.916	0.458	0.469	0.458	0.470
0.750	0.469	0.479	0.477	0.484
0.666	0.480	0.484	0.488	0.500
0.607	0.491	0.496	0.510	0.518
0.502	0.507	0.520	0.544	0.540
0.412	0.550	0.539	0.563	0.553
0.384	0.547	0.542	0.562	0.551
0.293	0.594	0.590	0.626	0.623
0.268	0.642	0.653	0.660	0.667
0.246	0.674	0.681	0.692	0.701
0.235	0.706	0.682	0.712	0.709
0.231	0.717	0.693	0.717	0.718
0.219	0.744	0.738	0.739	0.739
0.211	0.736	0.731	0.749	0.749
0.204	0.782	0.797	0.823	–
0.196	0.648	0.640	0.657	0.650
0.192	0.655	0.644	0.669	0.679
0.180	0.696	0.710	0.717	0.721
0.166	0.788	0.796	0.799	0.822

Cylinder 2"x18"

Channel Width 1.812 ft.

4.000	0.433	0.420	0.42	0.430
1.000	0.485	0.463	0.460	0.455
0.333	0.632	0.640	0.620	0.613
0.210	0.806	0.816	0.813	0.827
0.197	0.725	0.729	0.734	0.748
0.166	0.876	0.882	0.869	0.874

Channel Width 1.708 ft.

4.000	0.451	0.452	0.453	0.457
1.000	0.506	0.500	0.506	0.504
0.333	0.639	0.661	0.687	0.665
0.219	0.825	0.791	0.828	0.839
0.193	0.736	0.752	0.785	0.809
0.166	0.910	0.906	0.888	0.873

Channel Width-1.666 ft.

4.000	0.464	0.446	0.458	0.465
1.000	0.500	0.496	0.508	0.502
0.333	0.647	0.671	0.685	0.640
0.288	0.711	0.708	0.729	0.743
0.261	0.737	0.746	0.765	0.761
0.233	0.789	0.794	0.811	0.817
0.213	0.860	0.850	0.868	0.867
0.196	0.787	0.774	0.796	0.814
0.166	0.922	0.911	0.914	0.891

Channel Width-1.625 ft.

4.000	0.468	0.462	0.469	0.469
1.000	0.520	0.519	0.523	0.522
0.333	0.666	0.699	0.697	0.676
0.324	0.527	0.526	0.556	0.546
0.310	0.549	0.542	0.557	0.552
0.281	0.746 / 0.523	0.768 / 0.507	0.768 / 0.505	0.742 / 0.544
0.166	0.951	0.960	0.970	0.942

Channel Wall Interference

In wind channels the air stream is of limited extent, and restrictions are thereby imposed on the air flow past the model. Hence, in order to enable the results of wind channel tests to be applied to aircraft in an air space infinitely extended in all directions, the nature and magnitude of the influence exerted by the boundaries of the stream on the air flow, and resulting forces on the model, must be known.

Modern aerofoil theory has provided a basis for estimating the effect of the channel walls on the air flow past aerofoils, and the present study was made

Aeronautical Research Paper No.17. **TABLE Nº 2.**
UNIVERSITY OF TORONTO.
CHANNEL WALL INTERFERENCE. Aug. 25 1926
CYLINDER DRAG COEFFICIENTS.

Cylinder - 2"x18"

Channel Width - 1.583 ft.

Channel Height - ft.	Air Speed.			
	20	30	40	50
4.000	0.488	0.483	0.475	0.485
1.000	0.525	0.540	0.537	0.530
0.348	0.672	0.676	0.700	0.677
0.324	0.523	0.523	0.549	0.541
0.311	0.542	0.541	0.557	0.551
0.281	0.521	0.510	0.500	0.509
0.166	0.994	1.034	1.042	1.010

Channel Width 1.542 ft.

Channel Height - ft.	Air Speed.			
	20	30	40	50
4.000	0.525	0.529	0.529	0.523
1.166	0.573	0.573	0.582	0.583
0.500	0.678	0.678	0.695	0.679
0.419	0.732	0.739	0.741	0.720
0.379	0.759	0.759	0.768	0.749
0.344	0.565	0.562	0.580	0.574
0.310	0.584	0.591	0.610	0.606
0.281	0.507	0.506	0.510	0.513
0.251	0.820	0.803	0.810	0.826
0.243	0.758	0.748	0.715	0.712
0.230	0.869	1.113	1.111	1.079
0.203	1.118	0.959	0.900	1.149
0.170	1.516	1.507	1.533	1.520

Channel Width - 1.500 ft.

4.000	0.602	0.577	0.570	0.574
1.166	0.644	0.633	0.632	0.623
0.500	0.756	0.756	0.767	0.758
0.419	0.795	0.792	0.805	0.792
0.379	0.802	0.791	0.805	0.721
0.344	0.518	0.524	0.524	0.530
0.308	0.521	0.527	0.528	0.532
0.281	0.557	0.562	0.580	0.577
0.252	0.843	0.821	0.828	0.848
0.243	1.104 / 0.801	0.762	0.738	0.700
0.203	1.187	1.175	1.184	1.149
0.170	1.435	1.366	1.377	1.330

primarily with the object of securing further experimental confirmation of the effects predicted by theory.

There are two types of wind channel in general use. In one the air flow takes place through a closed tube and the air stream is thus limited by fixed walls parallel to the direction of flow. The air stream is of square, rectangular or circular cross section. In the other type the air flow takes the form of a free stream or jet of circular section without fixed boundaries.

The theory of the channel wall interference for an aerofoil in a circular section channel has been developed by Prandtl (Göttingen Nachrichten 1918-19, N.A.C.A. Report 116, N.A.C.A. Tech. Note No. 10) and the theory extended by

Aeronautical Research Paper No 17. **TABLE No 3.**
UNIVERSITY OF TORONTO.
CHANNEL WALL INTERFERENCE. Aug 25.1926
AEROFOIL COEFFICIENTS.
Aerofoil - 2 x 12

Lift Coefficient. **Air Speed - 20 fps.**

Incid deg	Channel Height - ft									
	4.00	1.75	1.50	1.00	0.500	0.333	0.250	0.166	0.125	0.104
-8	-0.1618	-0.1485	-0.1510	-0.1517	-0.1586		-0.1618	-0.1681		-0.1833
-4	-0.0489	-0.0467	-0.0430	-0.0474	-0.0500		-0.0872	-0.0999		-0.1574
0	+0.1513	+0.1466	+0.1554	+0.1504	+0.1536		+0.1681	+0.1485		+0.1175
+4	0.3385	0.3344	0.3236	0.3325	0.3698		0.4114	0.4228		0.3483
8	0.4535	0.4541	0.4602	0.4696	0.4766		0.4152	0.4216		0.4197
10	0.5186				0.4051		0.4165	0.4178		
12	0.4110	0.3862	0.3906	0.3950	0.3849		0.4216			
14		0.3773	0.3837	0.3653						

Drag Coefficient.

-8	.0581	.0521	.0474	.0525	.0556		.0600	.0676		.0689
-4	.0347	.0287	.0297	.0322	.0322		.0348	.0373		.0329
0	.0293	.0265	.0272	.0291	.0316		.0190	.0303		.0291
+4	.0360	.0326	.0360	.0367	.0367		.0329	.0386		.0493
8	.0502	.0502	.0506	.0556	.0537		.0657	.0777		.0828
10	.0651				.0758		.0853	.0916		
12	.0951	.0900	.0916	.0917	.0910		.1011			
14		.1024	.1093	.1043						

Lift Coefficient. **Air Speed - 30 fps.**

-8	-0.1612	-0.1543	-0.1556	-0.1610	-0.1598		-0.1691	-0.1667		-0.1845
-4	-0.0318	-0.0296	-0.0365	-0.0331	-0.0421		-0.0736	-0.0770		-0.1438
0	+0.1956	+0.1916	+0.1891	+0.1899	+0.2143		+0.2394	+0.2495		+0.2048
+4	0.3477	0.3502	0.3512	0.3480	0.3643		0.4253	0.4806		0.4430
8	0.4931	0.4778	0.4882	0.4803	0.4972		0.5685	0.6446		0.4441
10	0.5401				0.5770		0.4677	0.4469		
12	0.5949	0.5826	0.5806	0.4705	0.4486		0.4433			
14	0.4459	0.4237	0.4290	0.4098						

Drag Coefficient.

-4	.0525	.0490	.0525	.0520	.0539		.0559	.0615		.0660
-4	.0296	.0281	.0286	.0286	.0275		.0317	.0368		.0348
0	.0251	.0213	.0239	.0227	.0244		.0199	.0227		.0298
+4	.0293	.0289	.0298	.0292	.0267		.0222	.0270		.0416
8	.0483	.0427	.0441	.0444	.0388		.0360	.0433		.0786
10	.0551	.0655			.0494		.0739	.0854		
12	.0661	.0655	.0632	.0843	.0803		.0975			
14	.1042	.1014	.1020	.0994						

Glauert (R. and M. 723 and 867) to the cases of channels of square and rectangular sections.

In a channel with open working section the presence of the boundaries deflects the air stream downward at the model, so that for a given lift the measured angle of incidence and drag are both higher than for an unlimited stream; while in a channel of closed working section the stream is deflected upward and the incidence and drag are both measured too low.

The present research was confined to an investigation of the wall interference effects in wind channels of the closed tube type, of square and rectangular cross sections.

Aeronautical Research Paper No. 17. **TABLE No. 4.**
UNIVERSITY OF TORONTO.
CHANNEL WALL INTERFERENCE. Aug. 25 1926 J.M.P.

Aerofoil - 2-12
Lift Coefficient. Air Speed - 40 fps.

Incid. deg.	Channel Height - ft. 4·00	1·75	1·50	1·00	0·500	0·333	0·250	0·166	0·125	0·104
-8	-0.1671	-0.1654	-0.1694	-0.1683	-0.1653		+0.1800	-0.1716		-0.1909
-4	+0.0088	+0.0063	+0.0063	+0.0048	+0.0038		-0.0201	-0.0577		-0.1021
0	0.2047	0.2063	0.2088	0.2133	0.2277		+0.2577	+0.2890		+0.2814
+4	0.3448	0.3485	0.3535	0.3555	0.3783		0.4503	0.5045		
8	0.4799	0.4840	0.4917	0.4929	0.5265		0.6148	0.7038		
10					0.5880		0.5205	0.5007		
12	0.6009	0.6045	0.6113	0.6110	0.4743		0.4726			
14	0.4753	0.4702	0.4757	0.4675						
Drag Coefficient.										
-8	.0503	.0516	.0515	.0512	.0528		.0555	.0618		.0683
-4	.0269	.0278	.0289	.0278	.0276		.0295	.0325		.0341
0	.0215	.0193	.0202	.0194	.0185		.0182	.0197		.0242
+4	.0275	.0278	.0280	.0269	.0239		.0232	.0246		
8	.0421	.0404	.0412	.0397	.0363		.0349	.0378		
10					.0449		.0698	.0794		
12	.0626	.0600	.0643	.0629	.0820		.0926			
14	.0972	.0958	.0989	.0986						
Lift Coefficient. Air Speed - 50 fps.										
-8	-0.1630	-0.1734	-0.1661	-0.1669	-0.1720		-0.1816	-0.1688		-0.1881
-4	+0.0258	+0.0286	+0.0224	+0.0311	+0.0245		+0.0129	-0.0077		-0.0429
0	0.2068	0.2089	0.2135	0.2172	0.2298		0.2643	+0.2976		+0.3439
+4	0.3518	0.3524	0.3589	0.3589	0.3840		0.4505	0.5222		
8	0.4885	0.4862	0.5029	0.5004	0.5218		0.6112	0.7139		
10					0.6072		0.6913			
12	0.6115	0.6121	0.6236	0.6301	0.4921		0.4885			
14	0.6575	0.4884	0.4765	0.4750						
16	0.4412									
Drag Coefficient.										
-8	.0503	.0522	.0522	.0516	.0528		.0567	.0612		.0679
-4	.0239	.0254	.0233	.0251	.0248		.0276	.0289		.0320
0	.0192	.0184	.0197	.0190	.0166		0166	.0160		.0219
+4	.0255	.0259	.0252	.0249	.0224		.0187	.0207		
8	.0415	.0391	.0411	.0384	.0346		.0322	.0348		
10					.0419		.0410			
12	.0610	.0596	.0598	.0608	.0797		.0944			
14	.0728	.0962	.0973	.0992						
16	.1190									

PART I.

General Theory of an Aerofoil in an Unlimited Stream

(R. and M. 723, Engineering (Lond.), January 4, 1924).

According to the aerofoil theory, first suggested by Lanchester and developed in recent years by L. Prandtl and others at Göttingen, an aerofoil derives its lift from a circulation round the aerofoil which is superimposed on the motion of translation, resulting in an increase in velocity over the upper surface of the

Aeronautical Research Paper No17. **TABLE Nº 5.**
UNIVERSITY OF TORONTO.
CHANNEL WALL INTERFERENCE. Aug.25.1926
AEROFOIL COEFFICIENTS.
Aerofoil - 3x18.
Lift Coefficient. Air Speed - 20 fps.

Incid.	Channel Height - ft								
deg.	4.00	1.75	1.50	1.00	0.500	0.333	0.250	0.166	0.125	0.104
-8	-0.1700	-0.1677	-0.1638	-0.1641	-0.1596	-0.1753	-0.1725	-0.1731	-0.1942	-0.1989
-4	-0.0385	-0.0424	-0.0388	-0.0430	-0.0610	-0.0739	-0.0955	-0.1526	-0.1919	-0.2220
0	+0.1810	+0.1916	+0.1902	+0.1891	+0.2110	+0.2526	+0.2237	+0.2048	+0.1882	+0.1408
+2									0.3147	0.2372
4	0.3595	0.3430	0.3482	0.3619	0.3872	0.4333	0.4945	0.4999	0.3973	0.3181
6										0.3754
8	0.4502	0.4767	0.4754	0.4780	0.5386	0.6030	0.6448	0.4743		0.4305
10					0.4856	0.4557	0.4453	0.4743		0.4867
12	0.5716	0.5824	0.5886	0.4709	0.4440	0.4366	0.4156			
13	0.4695									
14		0.4356	0.4185							

Drag Coefficient.

-8	.0483	.0497	.0523	.0528	.0528	.0545	.0573	.0688	.0775	.0750
-4	.0295	.0301	.0301	.0281	.0295	.0326	.0334	.0365	.0486	.0458
0	.0222	.0216	.0197	.0203	.0216	.0222	.0239	.0315	.0371	0438
+2									.0450	0475
4	.0287	.0275	.0253	.0264	.0256	.0261	.0292	.0354	.0556	.0584
6										.0735
8	.0396	.0438	.0416	.0385	.0388	.0399	.0402	.0804		.0919
10					.0688	.0730	.0837	.0992		.1068
12	.0629	.0618	.0643	.0834	.0907	.0978	.1011			
13	.0854									
14		.1003	.0983							

Lift Coefficient. Air Speed - 30 fps.

-8	-0.1639	-0.1681	-0.1679	+0.1668	-0.1714	-0.1777	-0.1825	-0.1827	-0.1973	-0.1976
-4	+0.0046	+0.0030	+0.0060	+0.0022	-0.0101	-0.0193	-0.0532	-0.1010	-0.1237	-0.1382
0	0.2020	0.2065	0.2055	0.2119	+0.2366	+0.2654	+0.2847	+0.3183	+0.2507	+0.1627
+2									0.3194	0.2789
4	0.3470	0.3478	0.3563	0.3606	0.3985	0.4421	0.4950	0.6131	0.4247	0.3346
6										0.3656
8	0.4787	0.4811	0.4902	0.5039	0.5518	0.6092	0.6752	0.7963		0.4254
10					0.6226	0.5030	0.4976	0.5018		0.6019
12	0.6039	0.6035	0.6021	0.6105	0.4869	0.4694	0.4605			
14	0.6398	0.4769	0.4639	0.4537						
16	0.4140									

Drag Coefficient.

-8	.0492	.0533	.0496	.0524	.0546	.0572	.0593	.0673	.0722	.0777
-4	.0252	.0262	.0260	.0251	.0273	.0286	.0307	.0371	.0397	.0436
0	.0186	.0190	.0187	.0180	.0183	.0185	.0192	.0245	.0343	.0409
+2									.0422	.0471
4	.0252	.0241	.0256	.0231	.0227	.0217	.0223	.0286	.0522	.0589
6										.0690
8	.0386	.0398	.0392	.0362	.0330	.0328	.0332	.0474		.0936
10					.0418	.0704	.0805	.0903		.1102
12	.0579	.0576	.0591	.0558						
14	.0700	.0963	.0966	.0980						
16	.1168									

aerofoil, accompanied by a decrease in pressure and a decrease in velocity over the lower surface accompanied by a corresponding increase in pressure.

Flow conditions close to the aerofoil are modified by the skin friction between fluid and surfaces, and the two streams re-uniting at the trailing edge have, in general, different velocities, and are separated by an unstable sheet. The action of any small viscosity in the fluid will cause the sheet to develop into a thin region of vortex motion springing from the aerofoil in the immediate neighbour-

Aeronautical Research Paper No.17 **TABLE No 6.**
UNIVERSITY OF TORONTO.
CHANNEL WALL INTERFERENCE Aug 25 1926.
AEROFOIL COEFFICIENTS

Aerofoil - 3x18.
Lift Coefficient. Air Speed - 40 fps.

Incid. deg.	Channel Height - ft. 4.00	1.75	1.50	1.00	0.500	0.333	0.250	0.166	0.125	0.104
-8	-0.1640	-0.1684	-0.1676	-0.1696	-0.1746	-0.1844	-0.1867	-0.1895	-0.1970	-0.1981
-4	+0.0379	+0.0353	+0.0342	+0.0364	+0.0244	+0.0264	+0.0102	-0.0166	-0.0378	-0.0483
0	0.2062	0.2105	0.2134	0.2191	0.2380	0.2728	0.2979	+0.3651	+0.3929	+0.1925
+2										0.2672
4	0.3521	0.3599	0.3626	0.3718	0.4122	0.4606	0.5127	0.6555		
6										0.3646
8	0.4910	0.4962	0.4966	0.5140	0.5671	0.6301	0.6999	0.8562		0.4303
10					0.6314	0.7005	0.7569			0.4873
12	0.6175	0.6191	0.6234	0.6382	0.4981	0.4916	0.4695			
13	0.6432									
14	0.6687	0.4861	0.4897	0.4878						
16	0.4373									

Drag Coefficient.

-8	.0495	.0513	.0517	.0539	.0558	.0576	.0610	.0677	.0704	.0763
-4	.0219	.0235	.0232	.0231	.0253	.0256	.0285	.0325	.0358	.0455
0	.0177	.0181	.0178	.0171	.0164	.0169	.0157	.0199	.0262	.0373
+2										.0466
4	.0237	.0235	.0235	.0226	.0212	.0199	.0205	.0266		
6										.1154
8	.0377	.0382	.0377	.0356	.0323	.0312	.0310	.0477		.1363
10					.0396	.0391	.0404			.1544
12	.0592	.0576	.0569	.0547	.0834	.0964	.1019			
13	.0637									
14	.0685	.0961	.0986	.0970						
16	.1187									

Lift Coefficient. Air Speed - 50 fps.

-8	-0.1560	-0.1603	-0.1601	-0.1649	-0.1740	-0.1789	-0.1898	-0.1907	-0.1999	-0.2072
-4	+0.0508	+0.0513	+0.0505	+0.0505	+0.0470	+0.0548	+0.0482	+0.0347	+0.0375	+0.0276
0	0.2061	0.2134	0.2145	0.2241	0.2447	0.2802	0.3034	0.3805	0.4470	
+4	0.3528	0.3643	0.3643	0.3726	0.4160	0.4764	0.5361	0.7085		
8	0.4935	0.4967	0.5041	0.5218	0.5740	0.6521	0.7254			
10					0.6364	0.7153	0.7825			
12	0.6234	0.6242	0.6309	0.6397		0.5122	0.5063			
14	0.6705	0.6661	0.6678	0.5801						
18	0.4243									

Drag Coefficient.

-8	.0488	.0511	.0499	.0515	.0552	.0571	.0598	.0664	.0705	.0770
-4	.0207	.0216	.0215	.0212	.0225	.0229	.0246	.0293	.0276	.0421
0	.0165	.0164	.0167	.0160	.0157	.0151	.0154	.0207	.0254	
+4	.0227	.0232	.0232	.0220	.0198	.0195	.0201	.0297		
8	.0374	.0371	.0370	.0348	.0313	.0306	.0310			
10					.0385	.0380	.0387			
12	.0571	.0562	.0568	.0517		.0940	.1027			
14	.0679	.0677	.0662	.0899						
18	.1386									

hood of the trailing edge. The extent of the region depends on the form and attitude of the aerofoil. The drag, D_0,* due to these causes is termed the "profile drag," and for aerofoils of good shape may approach the skin friction drag. It is the only drag occurring for the wing of infinite span around which the flow is two-dimensional.

For the actual wing the flow is three-dimensional. The lift of a wing of finite span varies from a maximum at mid-span to zero at the tips, as a result of which a sheet of trailing vortices springs from the trailing edge of the wing, and

Aeronautical Research Paper No 17. **TABLE No 7.**
UNIVERSITY OF TORONTO.
CHANNEL WALL INTERFERENCE. Aug 25 1926.

Numerical Values of Quantities Appearing in Theoretical Relations for Channel Wall Corrections for Conditions of Tests.

Channel Characteristics						Aerofoil Area S=25C		$\delta\frac{S}{C}$		$573\delta\frac{S}{C}$	
Height h		Width b	$\lambda=\frac{h}{b}$	δ	Area C=hb	Aerofoil		Aerofoil		Aerofoil	
ins	ft.	ft			sq.ft.	2x12	3x18	2x12	3x18	2x12	3x18
48	4.000	4.00	1.0000	0.274	16.000	0.166	0.375	0.00285	0.00642	0.163	0.368
21	1.750	"	0.4375	0.305	7.000	"	"	0.00726	0.01634	0.416	0.936
18	1.500	"	0.3750	0.351	6.000	"	"	0.00975	0.02194	0.559	1.257
12	1.000	"	0.2500	0.524	4.000	"	"	0.0218	0.0491	1.249	2.813
6	0.500	"	0.1250	1.048	2.000	"	"	0.0873	0.1964	5.002	11.25
4	0.333	"	0.0833	1.560	1.333	"	"	0.1950	0.439	11.17	25.15
3	0.250	"	0.0625	2.090	1.000	"	"	0.348	0.785	19.94	44.98
2	0.166	"	0.0416	3.140	0.666	"	"	0.785	1.77	44.98	101.42
1.5	0.125	"	0.0312	4.190	0.500	"	"	1.396	3.14	79.99	179.9
1.25	0.104	"	0.0260	5.040	0.416	"	"	2.019	4.54	115.7	260.1

* A list of symbols is given at the end of the report.

being unstable, this sheet rolls up into two trailing vortices whose distance apart is slightly less than the wing span. Under their mutual reaction these two vortices will move slowly downwards and so will be inclined at a small angle to the wing. The effect of these trailing vortices is to give rise to a vertical velocity, w, at the aerofoil, termed the "induced velocity."

The induced velocity reduces the angle of incidence α by an amount w/V so that the lift force L_o for the effective angle of incidence α_o is inclined backwards at an angle w/V to the normal to the air stream direction, and possesses a drag component D_1 termed the "induced drag." The total drag of the aerofoil is the sum of the profile drag and the induced drag.

Assuming an elliptic distribution of lift across the span, the induced velocity is constant across the span and equal to

$$w = L/2\pi\rho V s^2$$

The relations between the angles of incidence of the finite and infinite span wings are then

$$\alpha = \alpha_o + (w/V) = \alpha_o + (K_L S/2\pi s^2) = \alpha_o + (2K_L/\pi A)$$

The induced drag of the finite wing is given by

$$D_1 = (w/V)\, L \text{ or } K_{D1} = (w/V)\, K_L \text{ and } K_{D1} = (K_L^2\, S/2\pi s^2) = (2K_L^2/\pi A)$$

and the relationship between the drags of finite and infinite wings are then

$$K_D = K_{Do} + K_{D1}$$

Extension of the Theory to the Case of an Aerofoil in a Limited Stream

The limited extent of the air stream in a wind channel places restrictions on the flow about the model and modifies the above results. The boundaries of the stream then give rise to induced velocities with corresponding deviations of the air stream.

Prandtl states that the boundary condition for a channel with a closed working section is zero normal velocity, and for the jet type channel with open working section, constant pressure. To secure these conditions for a channel of circular section, Prandtl combines with the aerofoil another obtained by reflection according to reciprocal radii. For a monoplane with assumed elliptic distribution of lift in the centre of the stream it is shown that the induced velocity due to the stream boundary at a distance, x, from the middle of the stream is

$$w^1 = (L/4\pi R^2 \rho V)\,[1 + (\tfrac{3}{4})\,\{(2s/R^2)\,x\}^2 + (\tfrac{5}{8})\,\{(2s/R^2)\,x\}^4 + (35/128)\,(\quad) + \ldots]$$

At the centre of the stream where $x = o$ the deviation of the air stream due to this induced velocity will be

$$w^1/V = \Delta\alpha = (\tfrac{1}{4})\,(S/C)\,K_L$$

The change in drag calculated from the above velocity is

$$D^1 = (L^2/4\pi R^2 \rho V^2)\,[1 + (3/16)\,(s/R)^4 + (5/64)\,(s/R)^8 + \ldots]$$

The first term in this equation is found to be the same for a uniform distribution of lift. Further, since with s/R as great as 0.6 the second term is less than 3 per cent. of the first, all terms except the first may be neglected. Prandtl has also found that the expression is the same for any small wing system in the centre of a circular stream. Hence, in general, for circular channels, the induced drag due to the channel wall is

$$D^1 = (L^2/4C\rho V^2)$$

$$\text{or } K_D{}^1 = (K_L^2\, S/4\pi R^2) = (\tfrac{1}{4})\,(S/C)\,K_L{}^2 = (K_L{}^2/\pi A)\,(s^2/R^2)$$

This is the extent to which the drag coefficient, as determined in a channel, will differ from that in an unlimited stream. For an open working section or jet $K_D{}^1$ is to be subtracted from the measured coefficient, and for a closed channel or tube is to be added to the measured coefficient to obtain the coefficient applicable to free air conditions.

The induced drag for the unlimited stream being

$$K_D{}^1 = (K_L{}^2 S/2\pi s^2) = (2K_L{}^2/\pi A)$$

the total induced drag in a stream of radius R is

$$K_{D1} + K_D{}^1 = (2K_L{}^2/\pi A)\ \{\ (1 \pm \tfrac{1}{2}\ \ (s^2/R^2)\ \}$$

where the plus sign applies to the open channel and the minus sign to the closed channel.

Wall Interference in Square and Rectangular Cross-Section Channels

The theory of channel wall interference has been extended to the cases of square and rectangular channels by H. Glauert (R. & M. 723 and 867).

In order to satisfy the boundary conditions for a closed type channel, there must be introduced a doubly infinite series of images corresponding to the "mirror" images to be seen if the channel walls were optical mirrors. The images are the same as the aerofoil, but alternate ones, commencing with the first, are inverted with respect to the aerofoil.

The case is first considered of an air stream having boundaries perpendicular to the span, but none above or below. There is only one series of images, all of which are identical with the aerofoil. The interference for this case, assuming elliptic distribution of lift and a line vortex system, is shown to be

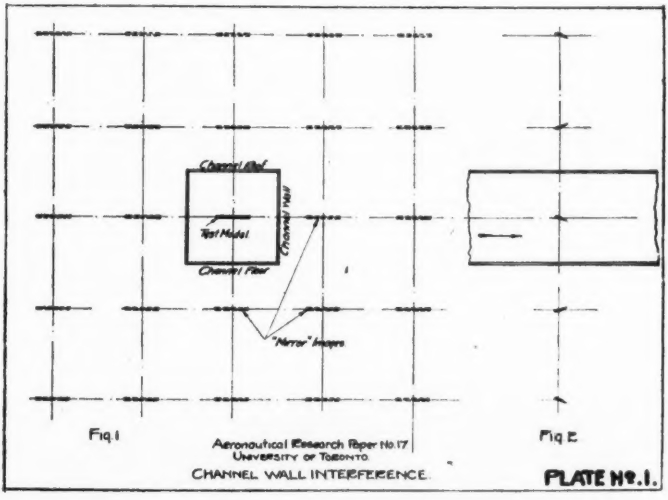
$$\Delta\alpha = (\pi S/12b^2)\, K_L = 0.262\ (S/b^2)\ K_L$$
$$\Delta\, K_D = K_L\, \Delta\, \alpha = 0.262\ (S/b^2)\ K_L{}^2$$

where b is the breadth of the channel.

In obtaining this expression, as before, all terms other than the first are regarded as of negligible importance for values of $(2s/b)$ as great as 0.6. The equation applies strictly at the centre of the channel, but the increase in interference from centre to tips is also negligible. The same expression applies for uniform distribution of lift.

For boundaries above and below the wing only, that is a channel of height, h, but of infinite breadth, the images are of alternate sign, and the resulting expressions for the interference are:

$$\Delta\, \alpha = (\pi S/24h^2)\ K_L = 0.131\ (S/h^2)\ K_L$$
$$\Delta\, K_D = K_L\, \Delta\, \alpha = 0.131\ (S/h^2)\ K_L{}^2$$



A comparison of the above expressions indicates that the interference is due mainly to the side walls of a rectangular channel, and not to the boundaries above and below the wing.

For a rectangular channel of breadth, b, and height, h, there is a doubly infinite series of images, and the interference is given by

$$\Delta\alpha = \delta\,(S/C)\,K_L \text{ in radians}$$
$$\Delta\alpha = 57{\cdot}3\,\delta\,(S/C)\,K_L \text{ in degrees}$$

where C = cross sectional area hb of a rectangular channel

$$\delta = \pi 2\,\lambda\left[(1/24) + \sum_{1}^{a} p/(1 + e^2 p\lambda\pi\right]$$

$$\lambda = (h/b)$$

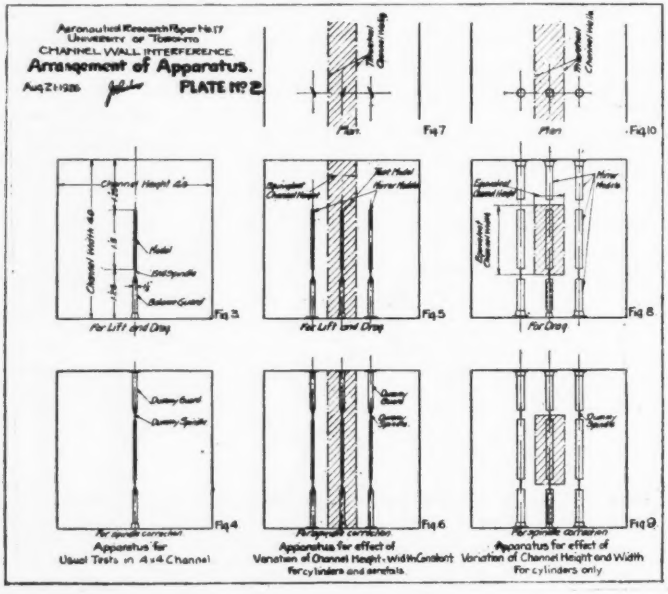
Numerical values of the coefficient δ for different values of the ratio $\lambda = (h/b)$ are plotted in Plate 3.

This curve appears to indicate that the interference of the walls on an aerofoil is a minimum for a rectangular channel having a breadth $\sqrt{2}$ times the height. There are also the curious results that δ is unaltered when λ is replaced by $\frac{1}{2}\lambda$ so that the interference is the same for a channel of breadth b and height h as for one of breadth $\sqrt{2}h$ and height $b/\sqrt{2}$.

The equation is applicable for aerofoils such that the span does not exceed 0.6 times the channel dimension parallel to the span. If this figure is exceeded, the interference will be greater than that given by the equation.

The formula can also be used for biplanes, since to a first approximation both wings can be regarded as at the centre of the channel, and the strength of the images is therefore doubled, which is taken account of automatically in the formula in the symbol S.

It should be noted that the interference according to the above equations depends on the area of the wing, and not on the chord or span independently.

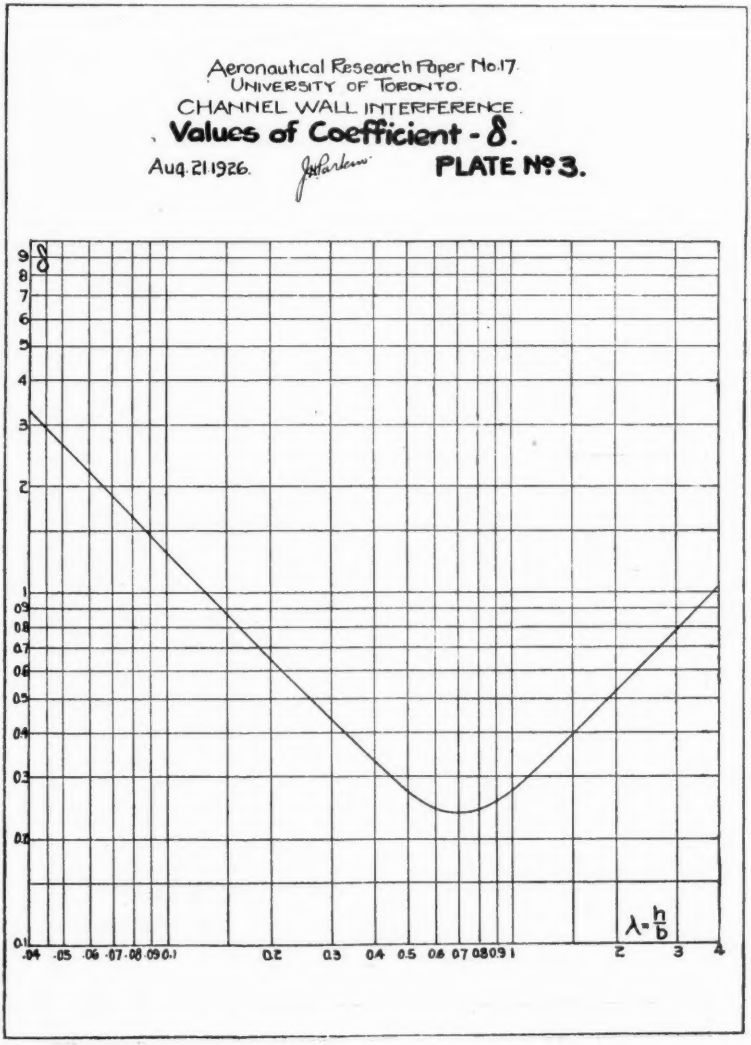

Aeronautical Research Paper No 17
University of Toronto
CHANNEL WALL INTERFERENCE.
Arrangement of Apparatus.
PLATE No 2.
Fig 3
Fig 4
Fig 5
Fig 6
Fig 7
Fig 8
Fig 9
Fig 10
Apparatus for Usual Tests in 4 x 4 Channel
Apparatus for effect of Variation of Channel Height - Width Constant For cylinders and aerofoils.
Apparatus for effect of Variation of Channel Height and Width For cylinders only

The percentage interference will, however, be less the smaller the aspect ratio, since the induced drag increases with decrease in aspect ratio.

PART II.

Method of Test

Experimental confirmation of the accuracy of the wall interference corrections according to the Prandtl theory is difficult (Glauert, Int. Air Congress, 1923, p. 254) and has usually been made by testing the same aerofoil in channels of different sizes (R. & M. 889 and 898) or testing geometrically similar aerofoils of different sizes in the same channel.



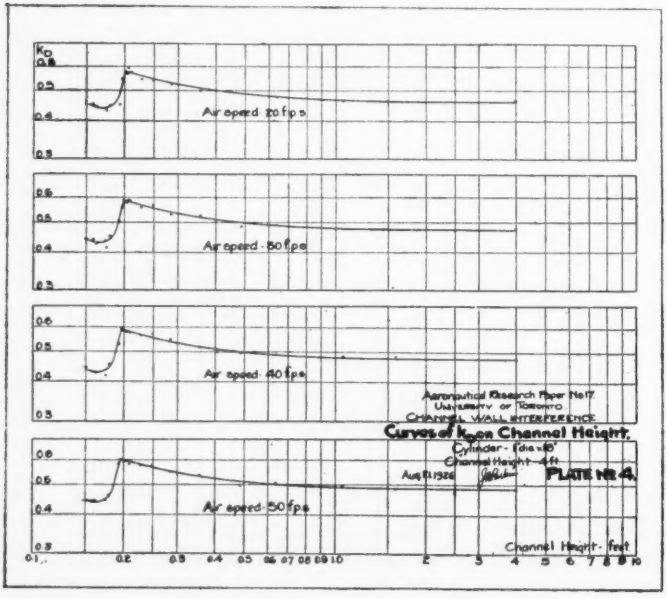
For a closed working section channel the boundary condition of zero normal velocity is secured analytically by the introduction of a system of images of the test aerofoil, obtained by regarding the channel walls as optical mirrors. The test aerofoil is thus the centre of a system of mirror images of itself, as shown in Figs. 1 and 2, Plate 1. In the method of test employed in this research, which was suggested by Colonel E. W. Stedman, R.C.A.F., F.R.Ae.S., models are introduced in the channel to act as the first images adjacent to the test model, as indicated in Plate 2. The model under test is then in effect in a wind channel of dimension equal to one-half the distance between two corresponding image or mirror models (Figs. 5 and 7, Plate 2). The method has been termed the " mirror " method and is analogous in principle to the reflection method previously used in determining the ground influence on aerofoils (N.A.C.A. Tech. Note No. 67, 1921).

The mirror method possesses several points of superiority compared with methods so far used for studying channel wall interference :

1. The measurements are made throughout on one and the same model. This effectually eliminates any effects due to slight differences in geometrical similarity of the models, roughness or nature of surfaces, etc., that may occur if models of different sizes are used.
2. The nature of the general air flow in the channel is the same for all the tests, hence variations in results due to differences in turbulence, velocity distribution, etc., which may be present when the same model is tested in different channels are obviated.
3. The method in effect provides an elastic channel whose cross sectional dimensions can be varied at will, within wide limits.

On the other hand :

1. A number of exactly similar models are required.
2. Only the first " mirror " images are used. These in turn have images outside the channel walls, and the influence of the actual channel walls on the air flow about the " mirror " models may affect the results.


Air speed 20 f.p.s.
Air speed 30 f.p.s.
Air speed 40 f.p.s.
Air speed 50 f.p.s.
Aeronautical Research Paper No 17
University of Toronto
CHANNEL WALL INTERFERENCE
Curves of k_D on Channel Height.
Cylinder - Dia. 6"
Channel Height - 4 ft.
Aug 11 1926
PLATE No 4.
Channel Height - feet

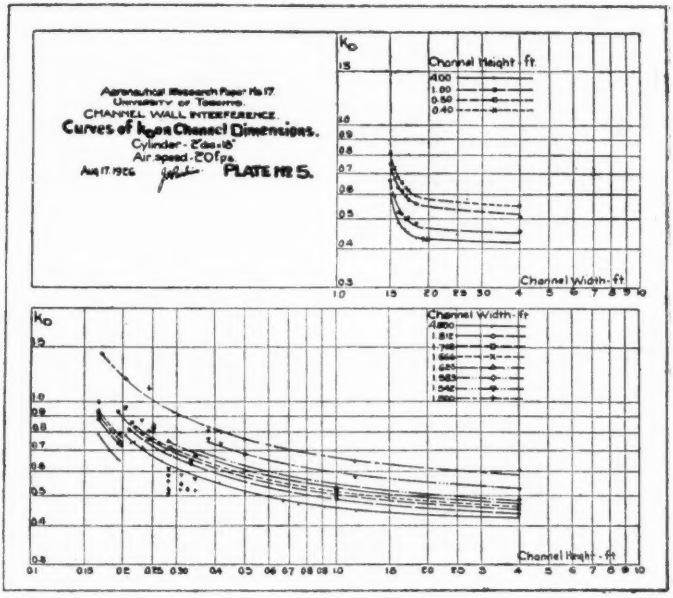
Wind Channel Employed

The tests were carried out in the new R.A.E. type 4ft. wind channel of the Aerodynamic Laboratory of the University of Toronto.* The aerodynamic forces were measured on an N.P.L. type balance employing the usual vertical steel spindle method of support of the models. Air speeds were controlled manually and indicated by side plate and modified Chattock micromanometer.

Models

Preliminary tests were made on cylinders for the following reasons:

1. Simple geometrical shape and low cost.



2. Permit the use of mirror models at the ends of the test model, since the mirror model may surround the balance standard which is not possible with aerofoils of usual form. This permits the study of the effect of variation of channel width as well as height.
3. Absence of cross wind force simplifies measurements somewhat and also provides data on wall interference for symmetrical models on which only drag is exerted.

The cylinders employed were of selected drawn 18 gauge brass tubing, 18in. long and of two outside diameters, 1in. and 2in. Brass plugs were fitted in the ends of the cylinders to provide plain ends normal to the axis and also to take the 5/16in. diameter steel spindles.

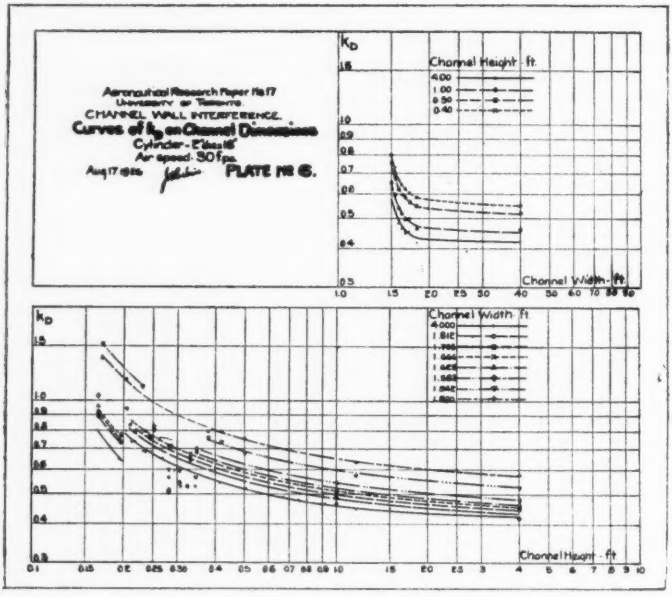
The aerofoil section employed was that known as the Airscrew 4 (R. & M. 322, Bairstow—Applied Aerodynamics, p. 304, Aero Research Paper No. 8, Toronto) and was selected for the following reasons:

* For a description of the channel and apparatus, see Aeronautical Research Paper No. 16.

1. It is a comparatively simple section, with flat undersurface and good efficiency.
2. It is a moderately thick, $t/c=0.127$, high lift section. The higher the lift, the greater the wall interference effects, hence the advantage in using a high lift section.
3. Deflection difficulties are lessened with a rigid thick section.

The aerofoil models were made by W. H. Nichols, of Waltham, Mass., on a special wing generating machine (N.A.C.A. Tech. Note No. 172) which produces aerofoils guaranteed accurate within 0.002in. on any ordinate. The models were of duralumin.

Two sets of aerofoil models were tested, one of 2in. chord and 12in. span, the other of 3in. chord and 18in. span, in order to check the effect of ratio of



model to channel size and of vl effect, and also to ensure obtaining general conclusions.

The larger models were supported on very slightly tapered 5/16in. dia. steel end spindles, the smaller ones on spindles of somewhat greater taper. All spindles fitted in accurately reamed and threaded holes in the aerofoils.

The mirror models were supported on similar end spindles held in by set screws in the dummy balance guards (or end mirror cylinders). The latter in turn were held in sockets fitted to sliders. The sliders were arranged with tapered clamping strips enabling them to be clamped in any position on steel rails. The steel rails were sunk in the floor and roof of the channel, extending perpendicularly to the air stream from side to side of the channel directly in line with the balance axis. This enabled the mirror models to be accurately located with reference to the test model. The general arrangement of the apparatus will be evident from the photographs.

PART III.

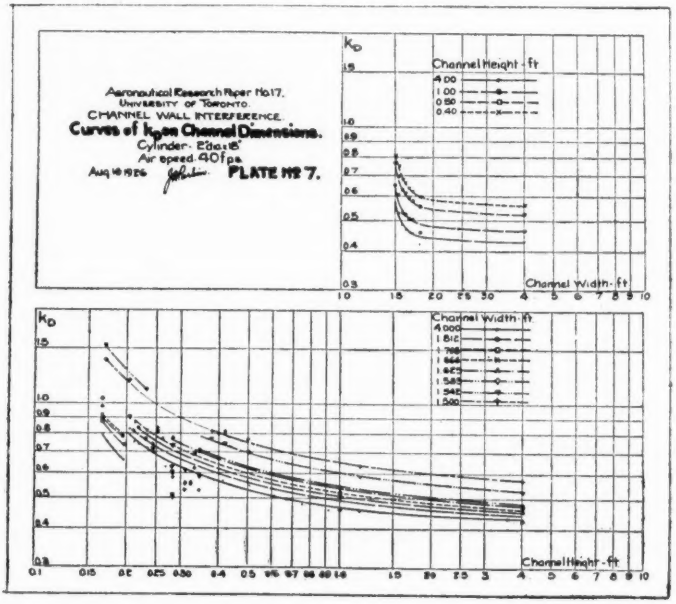
Tests on Cylinders

There were two series of tests made on the cylindrical models.

Series I. Resistance Measurements.

Measurements of drag were made on cylinders under the following conditions:

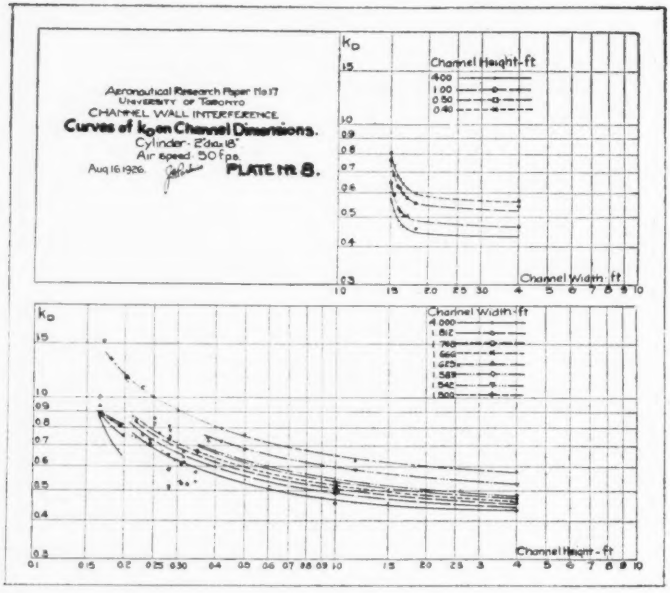
1. For channels of various heights, from a maximum equal to 4ft. to the minimum possible with the apparatus 0.15 to 0.166 ft., the channel width being constant at 4ft. In these tests two mirror cylinders only were used, their arrangement being as shown in Figs. 5 and 6, Plate 2. Figs. 1 and 2 show the special arrangement employed for a channel height equal to 4ft. The diagrams indicate the method used in determining spindle and guard corrections. These tests were made on both 1in. and 2in. diameter cylinders.



2. For channels of various heights and various widths, the arrangement of the cylinders is shown in Figs. 8, 9 and 10 and Photograph 1, eight mirror cylinders being used. The mirror cylinder surrounding the guard tube of the balance was provided with an end plug having a hole somewhat larger than the spindle to provide clearance. The presence of the balance standard and guard tube within this cylinder limited the maximum channel width that could be tested in this way, as it was thought to be undesirable to permit these to project beyond the end of the mirror cylinder. The range of channel widths tested was thus from 1.5ft. (equal to cylinder length) to 1.812ft. using the set up shown in Figs. 8 and 9, with the set up of Figs. 5 and 6 a special or limiting case for a width of 4ft. The range of heights tested was as above. These tests were made on the 2in. diameter cylinders only.

The spacing of the mirror cylinders was made as accurately as possible using micrometers. The final check on the accuracy of the setting was the absence of any cross wind force as shown by the balance. This check was very sensitive, a very small error in spacing causing the test cylinder to move to one side or the other, to the limit of its travel.

The drag of the test cylinder was measured for each set up at nominal air speeds of 20, 30, 40 and 50 feet per second. The projected or frontal area of the 2in. cylinders, test and mirror in the extreme case (see Photo 1) totals 2 sq. ft. out of a total channel cross section of 16 sq. ft. The nominal air speed upstream from the cylinders is hence increased at the cylinders in the proportion of 16 : 14. The actual air speed past the test cylinder is, however, probably quite different to either of these speeds. Hence, it was considered advisable to use the nominal channel air speed in calculating the coefficients, regarding any increase in speed due to presence of model as part of the channel effect.



Series II. Pressure Distribution Measurements.

In seeking an explanation of the peculiar variation in drag discovered in the Series I. tests, an investigation of the distribution of pressure around the 2in. × 18in. cylinder was made for typical channel heights and widths.

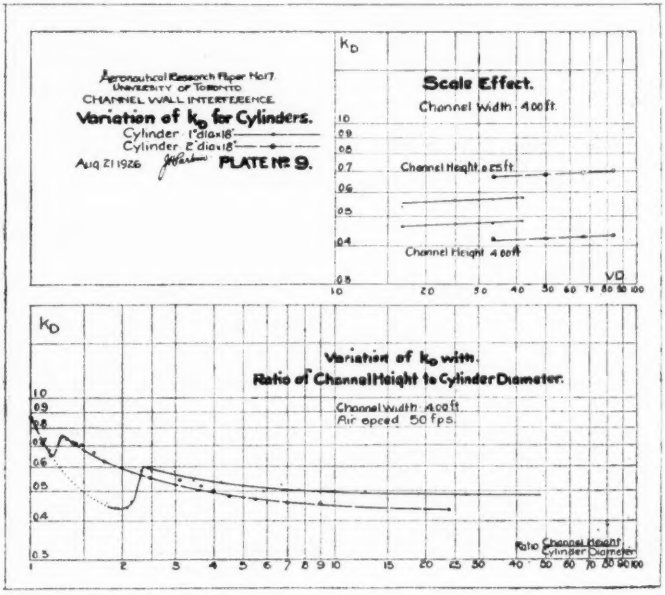
The test cylinder was fitted with airtight end plugs, the lower one carrying the spindle and also a small diameter pressure connection quite close to the spindle, from which a fine rubber tube passed to a micromanometer. The pressure on the surface of the cylinder was communicated to the interior by means of a 0.02in. diameter hole carefully drilled radially through the cylinder wall. Two such holes were drilled, one midway between the two ends of the cylinder, and the other 1.26in. from the upper end. The hole not in use was plugged with plasticene.

The pressure measured was the difference between that at a given point on the surface of the cylinder and that at the side plate in the channel wall. It was found that the difference between the pressure at the side plate and the static pressure in the centre of the channel in the position occupied by the cylinder was so small that for the purposes of this test the pressure at the side plate could be regarded as the static pressure at the cylinder. The pressure difference measured was thus that between the actual pressure and the normal static pressure at the same point on the cylinder surface.

The pressure difference was measured on a Krell slanting manometer previously calibrated against a Chattock micromanometer.

The cylinder was supported on an end spindle on the N.P.L. balance in the usual way, and the balance protractor used to enable the cylinder to be rotated through the proper angle to bring the pressure opening into any desired position. The pressures were thus measured at angular intervals of 10° for the full 360°.

The air speed was 40 f.p.s. throughout.



Results

Series I.—The results of the drag measurements in cylinders have been presented in the non-dimensional coefficient

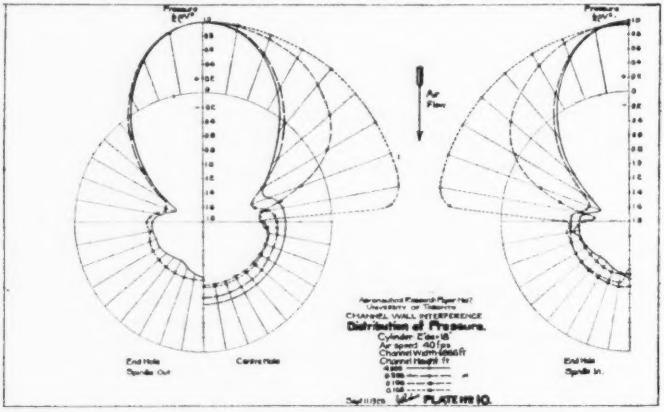
$$K_D = (D/\rho S V^2)$$

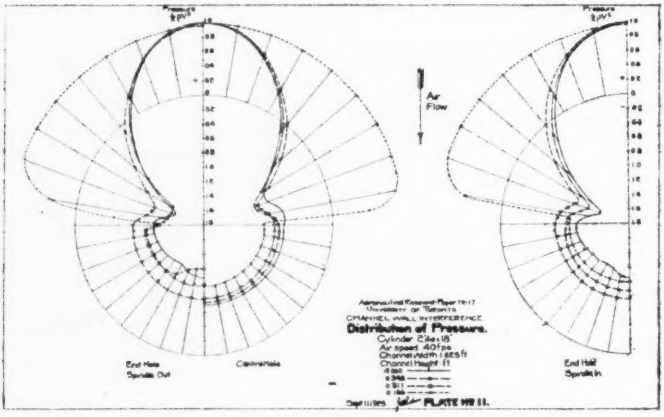
where K_D is the drag coefficient.

D is the measured drag in pounds corrected for spindle interferences.
ρ is the kinematic viscosity of air.
S is the projected area of the cylinder, *i.e.*, diameter × length in sq. ft.
V is the air speed in ft. per sec.

The coefficients are tabulated in Tables I. and II. and plotted in Plates 4-9.

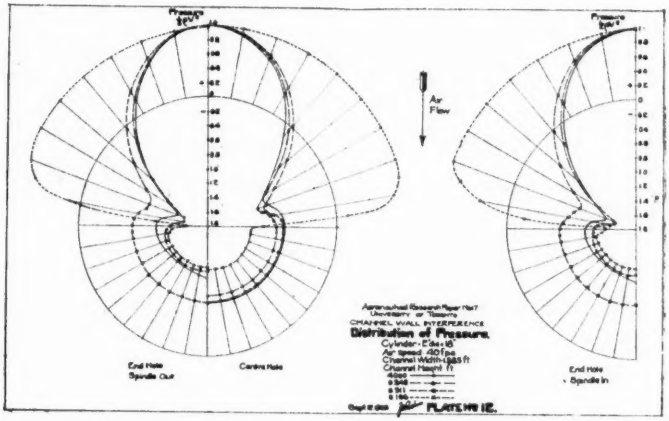
Series II.—The pressures observed in the pressure distribution tests on the cylinders have also been expressed non-dimensionally in terms of normal impact pressure, the observed pressures being divided by $\frac{1}{2}\rho V^2$. Due to the very large number of the pressure observations and also to the fact that the individual readings are of relatively minor importance, these results have not been tabulated, but are presented graphically in Plates 10-14.

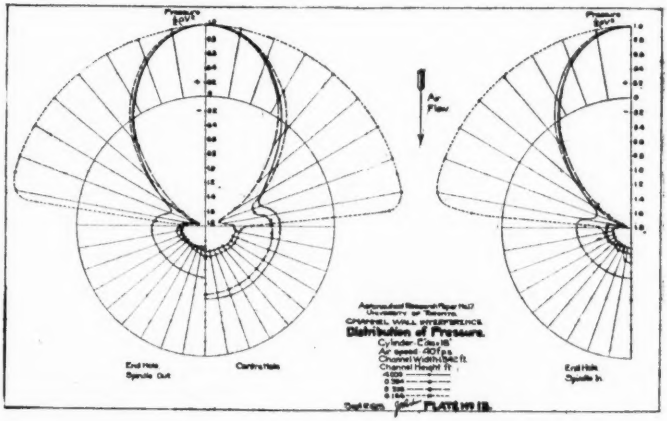


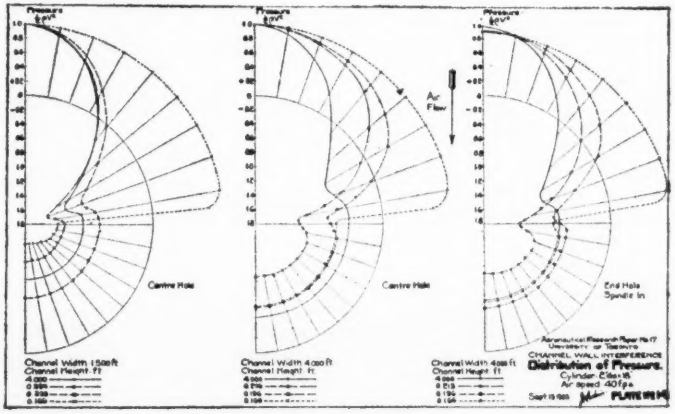


Discussion of Results

Series I.—The cylinder drag coefficients are plotted on Plates 4-9, logarithmic scales being used because of the wide range covered and the closeness of the readings at lower end of range. Plate 4 shows the variation of drag coefficient with channel height for different speeds for the 1in. × 18in. cylinder. On Plates 5-8 are plotted drag coefficients for the 2in. × 18in. cylinders, each plate carrying the results for one speed. In the lower part of each of these plates are drawn curves showing the variation of the drag coefficient with channel height for a number of channel widths. These curves are drawn from the plotted observations. In the upper part of each plate are plotted the drag coefficients as observed, or


Air Flow
End Hole Spindle Out
Centre Hole
End Hole Spindle In
Aeronautical Research Paper No 17 University of Toronto
CHANNEL WALL INTERFERENCE
Distribution of Pressure.
Cylinder C/dia 1/6
Air speed 401 f/s
Channel Width 1.665 ft
Channel Height ft
4.000
0.340
0.311
0.196
PLATE No 16.


Air Flow
End Hole Spindle Out
Centre Hole
End Hole Spindle In
Aeronautical Research Paper No 17 University of Toronto
CHANNEL WALL INTERFERENCE
Distribution of Pressure.
Cylinder C/dia 1/6
Air speed 401 f/s
Channel Width 1.162 ft
Channel Height ft
4.000
0.384
0.338
0.196
PLATE No 15.

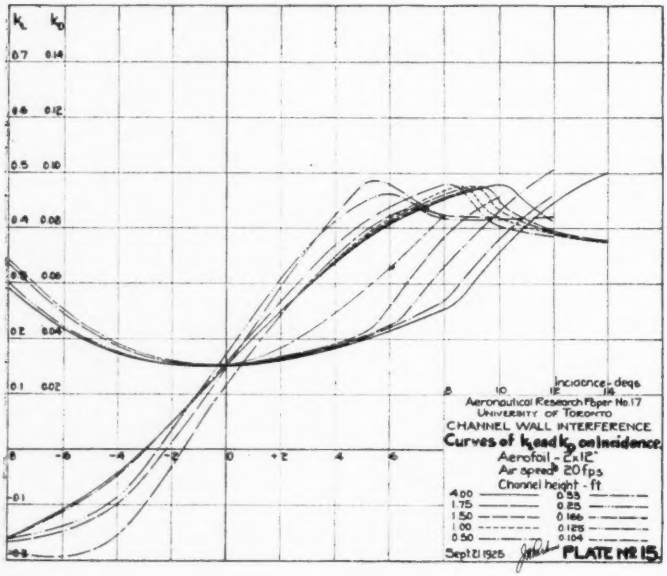

Air Flow
Centre Hole
Centre Hole
End Hole Spindle In
Channel Width 1.568 ft
Channel Height ft
4.000
0.340
0.311
0.196
Channel Width 4.000 ft
Channel Height ft
4.000
0.376
0.196
0.196
Channel Width 4.000 ft
Channel Height ft
4.000
0.213
0.196
0.196
Aeronautical Research Paper No 17 University of Toronto
CHANNEL WALL INTERFERENCE
Distribution of Pressure.
Cylinder C/dia 1/6
Air speed 401 f/s
PLATE No 14.

where there was no observation, taken from the curves in the lower part of the plate, on a channel width base. The resulting curves show the variation of the coefficient with channel width for different constant channel heights.

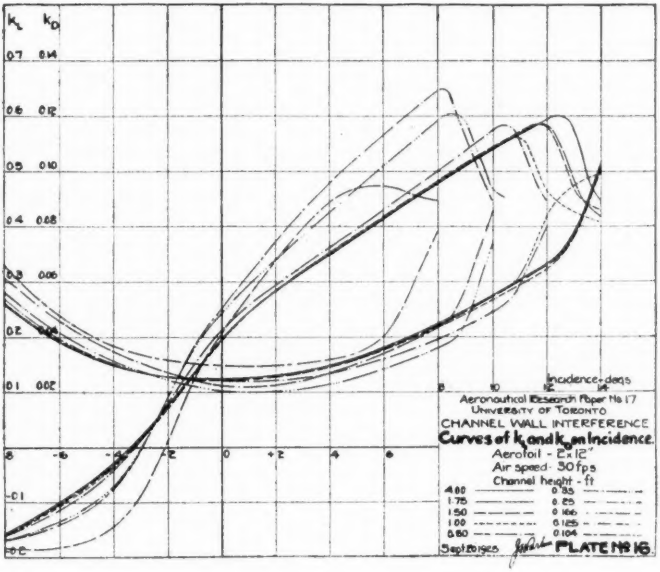
An examination of the curves of variation of drag coefficient K_D discloses a number of interesting features:

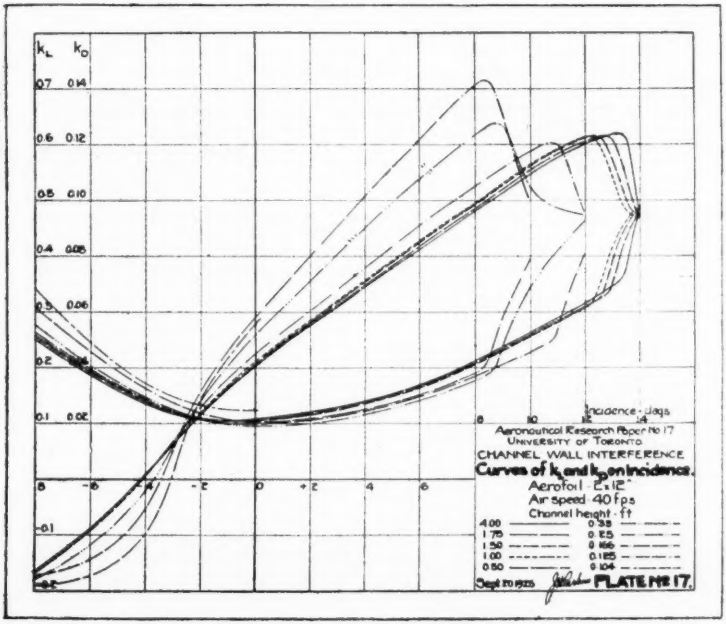
1. The drag increases with decrease in channel height and channel width. The increase is relatively less for the 1in. than for the 2in. cylinders. The increase in drag for a channel of height 0.2ft. as compared with that for a channel of height 4ft. is about 25 per cent. for the 1in. cylinder, while for the 2in. cylinder for channel widths as small as 1.666ft. the corresponding increase is about 100 per cent.

   Plotting K_D on ratio of channel height to cylinder diameter, as in Plate 9, indicates, however, that the increases in both cases are of the same general nature. The reason that the curves for the 1in. and 2in. cylinders do not exactly coincide in this plotting is probably due to the difference in the ratio of length to diameter of the two cylinders and resulting end effects.



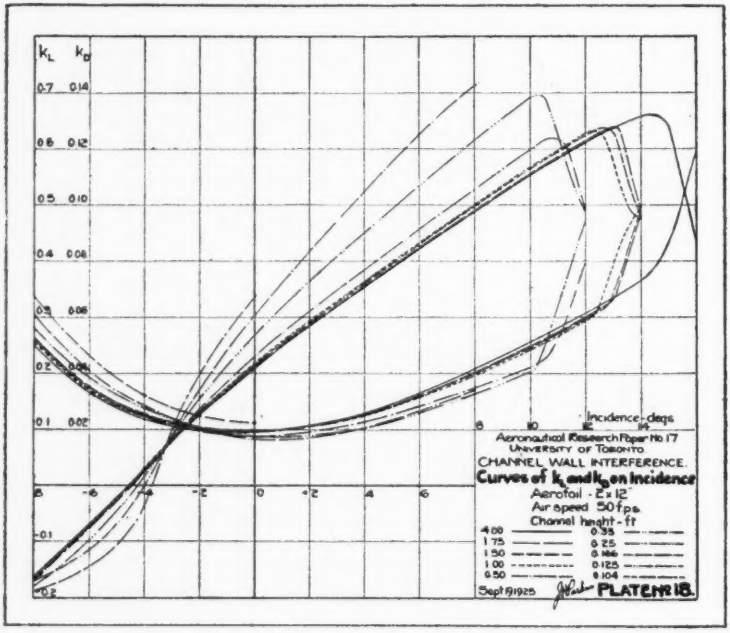
2. There is a region of instability, or critical point, encountered as the channel height is decreased where the drag abruptly decreases, and beyond which conditions are unstable for a greater or less range, afterwards becoming stable again and the drag increasing. In the case of the 1in. cylinders the drag drops at the critical point, a channel height of 0.2ft., to less than that for the 4ft. channel. The readings in the unstable region are generally steadier than for the larger cylinders, and with further reduction the drag tends to increase. It was impossible to make tests with the 1in. cylinders at channel heights as small relatively as for the 2in. cylinders, owing to the size of the balance guard tube, but the curve through the critical region shows the same upward tendency as for the larger cylinder,


k_L k_D
Incidence - degs
Aeronautical Research Paper No 17
University of Toronto
CHANNEL WALL INTERFERENCE
Curves of k_L and k_D on Incidence.
Aerofoil - 2"x12"
Air speed - 30 fps
Channel height - ft
4.00 0.135
1.75 0.125
1.50 0.166
1.00 0.125
0.50 0.104
Sept 29 1925
PLATE No 16.


k_L k_D
Incidence - degs
Aeronautical Research Paper No 17
University of Toronto
CHANNEL WALL INTERFERENCE
Curves of k_L and k_D on Incidence.
Aerofoil - 2"x12"
Air speed - 40 fps
Channel height - ft
4.00 0.135
1.75 0.125
1.50 0.166
1.00 0.125
0.50 0.104
Sept 29 1925
PLATE No 17.

which is borne out by the plotting on Plate 9. For the 2in. cylinders the curves of K_D variation beyond the unstable region appear to be of the same curvature as that before the break, but displaced downward. The displacement appears to become less as the channel width decreases, and for the minimum width the curves before and after the unstable region appear to be part of the same curve. It is therefore surmised that if the channel width could be reduced sufficiently to prevent air flow past the cylinder ends that the instability would disappear completely.

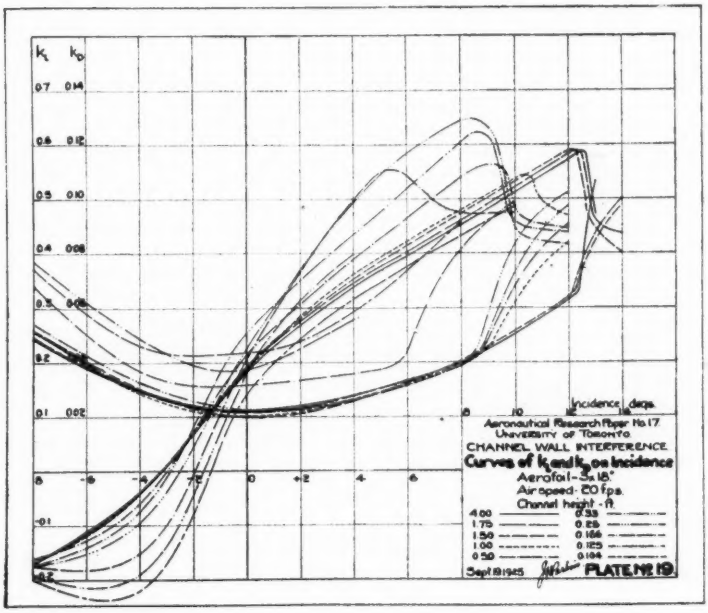
For channel widths from 4.0 to 1.666 ft. (2in. cylinders) the unstable region is very narrow, a reduction of a small fraction of an inch being sufficient to cause a reduction of about 20 per cent. in drag, and the instability is confined to this narrow region. For channel widths less than 1.666ft. the instability occurs for larger and larger channel heights, until

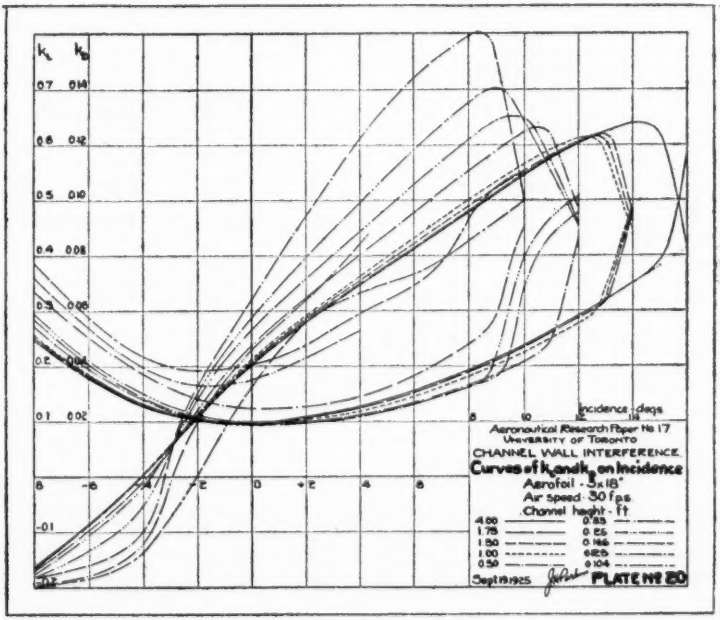


for the minimum channel width the unstable region extends from a height of 0.38 to about 0.2 ft. As the extent of the region increases, the unsteadiness and reduction in drag within it also increases, until with the minimum channel width the drag in this region is much less than for the 4ft. channel (compare 1in. cylinder). The unstable region is narrower and more clearly defined the greater the channel width.

The increase in the unstable region corresponds to the reduction in the critical angle for aerofoils with reduction in channel size. (See later.)

Within the unstable region readings are exceedingly difficult to secure, and several different drags may be indicated for the same conditions, as shown by the points plotted in this region on the graphs. Beyond the unstable region, *i.e.*, for the region between a channel height of 0.2 and

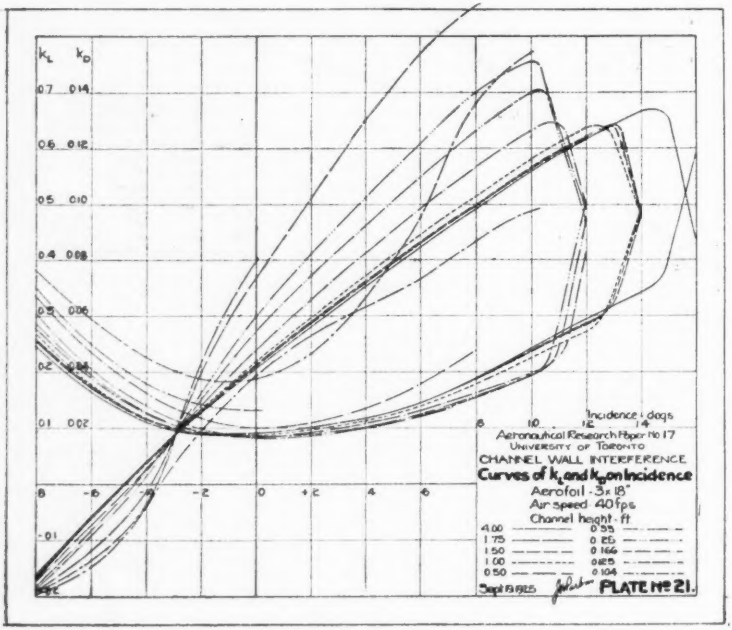

Incidence degs
Aeronautical Research Paper No 17
University of Toronto
CHANNEL WALL INTERFERENCE
Curves of k_1 and k_2 on Incidence
Aerofoil - 3x18"
Air speed - 20 fps
Channel height - ft
4·00 0·33
1·75 0·25
1·50 0·166
1·00 0·125
0·50 0·104
Sept 19 1925
PLATE Nº 19


Incidence degs
Aeronautical Research Paper No 17
University of Toronto
CHANNEL WALL INTERFERENCE
Curves of k_1 and k_2 on Incidence
Aerofoil - 3x18"
Air speed - 30 fps
Channel height - ft
4·00 0·33
1·75 0·25
1·50 0·166
1·00 0·125
0·50 0·104
Sept 19 1925
PLATE Nº 20

one of 0.166 (for 2in. cylinders) conditions are fairly stable, and most of the curves can be plotted here with reasonable definiteness. The maximum drag occurs for the minimum channel width.

3. The variation of drag with channel width, as shown by the curves for the 2in. cylinder, is of much the same character as that with channel height, but rather less proportionately until a width of about 2ft. is reached. Between 2 and 1.5 ft. the increase in drag is very rapid, averaging about 40 per cent.

4. The curves enable the increase in measured drag of a cylinder in a channel of small dimensions to be estimated. Thus, a 2in. × 18in. cylinder in a 2 × 2 channel would have about 10 per cent. greater drag than in a 4 × 4 channel, and about 15 per cent. greater drag in a 1 × 2 channel.



5. The curves plotted in the upper part of Plate 9 show the general nature of the scale effect on cylinder drag. The drag is seen to increase with speed or VD. This is in agreement with the general curve of scale effect for cylinders, given in R. & M. 102, 1913-14.

There is some slight indication that the scale effect is reduced the smaller the channel height and width, since the curves for a channel width of 1.5 ft. on Plates 5-8 practically coincide, as do those for a channel height of 0.40 ft. But the differences are so small as to be within the limits of experimental error.

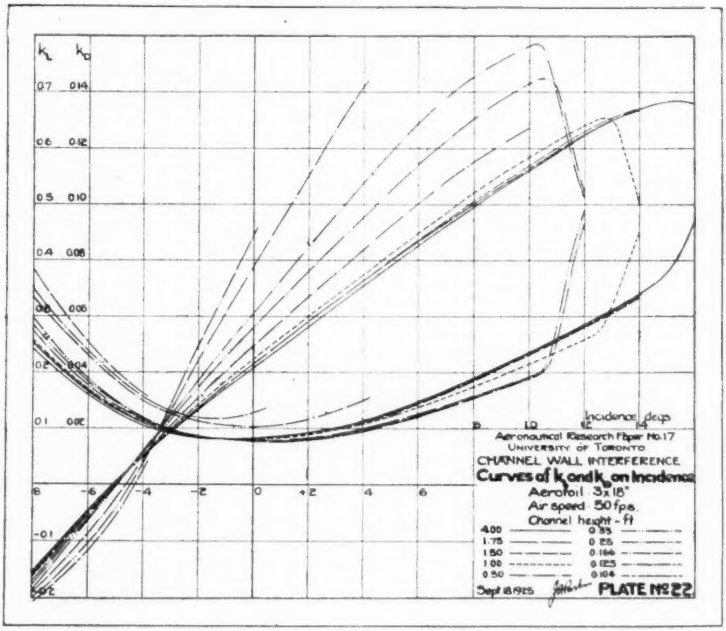
The critical point is influenced by speed to some extent. Thus, an examination of Plates 4-8 shows that the critical point occurs at a smaller channel height the higher the speed. The unstable region is reduced by increase in speed. This is in conformity with the observed scale effect on the critical angles for aerofoils, which are increased with speed.

In R. & M. 106, 1913-14, the measured drag per foot run on a 2in. cylinder, with cylindrical end guards, at 30 f.p.s. is 0.191 lbs., equivalent to 0.287 for the 18in. length.

This condition corresponds to that in the present research for a channel 4ft. high and 1.5ft. wide for which the observed drag was 0.308 lbs.

In R. & M. 102 a curve is given for the resistance of smooth wires. For a value of $\log (vl/\nu)$ equal to 4.5 (corresponding to the 2in. cylinder at 30 f.p.s.) the value of $(F/\rho V^2 L^2)$ shown is about 0.59, from which the resistance per diameter length is $F = 0.035$ lbs., and for the 18in. length the drag is 0.315 lbs.

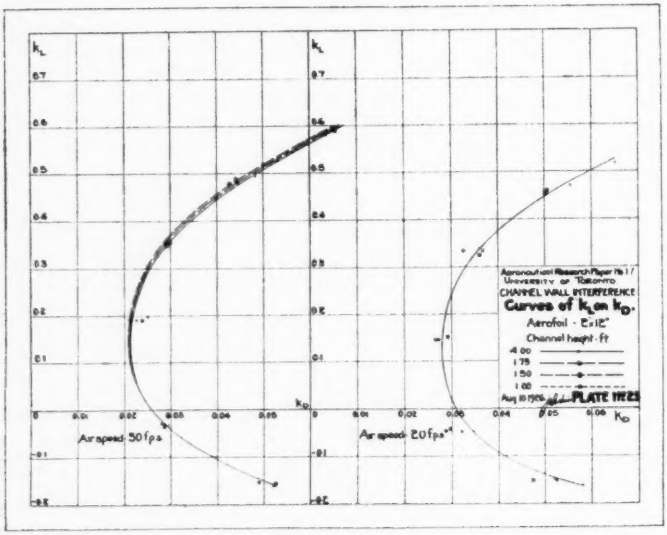
The difference in these results is probably due to the different flow texture or turbulence of the air stream in the different channels. (See N.A.C.A. Report 231 and Tech. Note No. 191.)

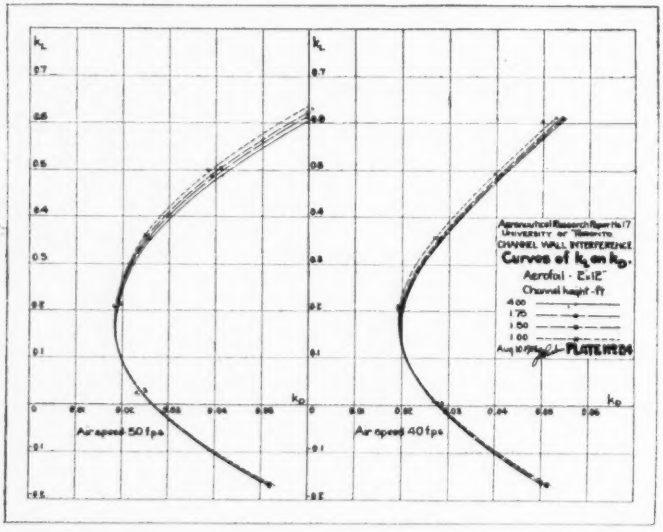


The critical point in the curves of K_D on channel height for both 1in. and 2in. cylinders for the larger channel widths occur at a height of about 0.2ft. The corresponding absolute dimension of the air gap between test and mirror cylinders is 0.4in. for the 2in. and 1.4in. for the 1in. cylinder. Again plotting K_D on ratio of channel height to cylinder diameter (Plate 9) the critical point is seen to occur at quite different values of the ratio. There is not sufficient information to determine what factors control the critical region. These tests indicate that:

1. It is lessened, *i.e.*, occurs for smaller channel heights the higher the speed.
2. It is increased the narrower the channel (Plates 5-8). It is also considered probable that the critical region depends on:
3. Absolute dimension of the gap between models.
4. Shape of the gap, ratio of length to width.
5. The curvature of the boundary surfaces of the gap.
6. End effects, *i.e.*, ratio of length to diameter of boundary models.

Series II.—Pressure distribution curves have been plotted in Plates 10-14. In the case of Plates 10-13 each plate shows the pressure distribution for one channel width. The figures plotted are the averages of the observed pressures at corresponding points on each side of the cylinder. The averages were used

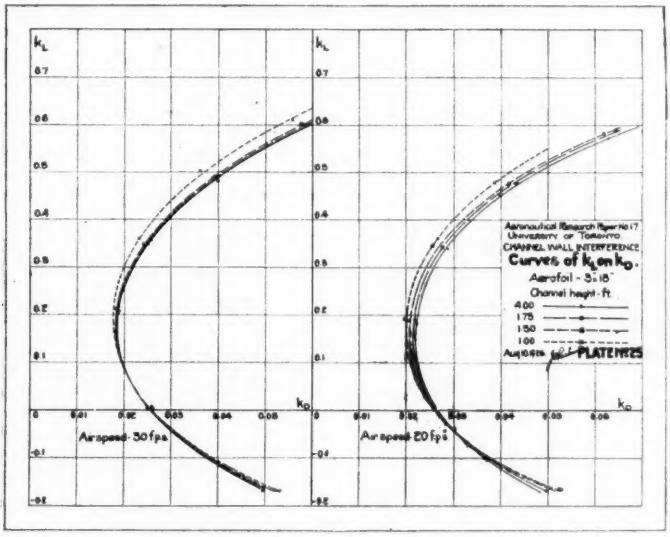


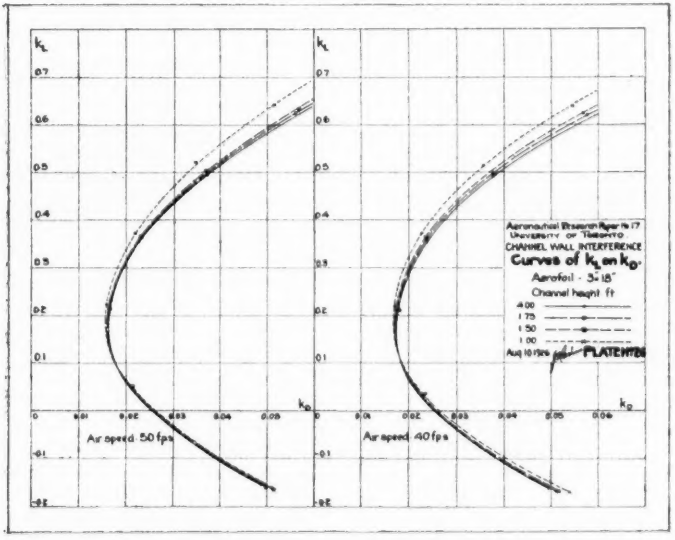


owing to the difficulty of initially setting the pressure opening exactly upstream. The average figures are plotted for the semi-cylinder only.

The central group of plottings on each plate is that for the central hole, *i.e.*, the pressure distributions in the median plane of the cylinder, 9in. from either end.

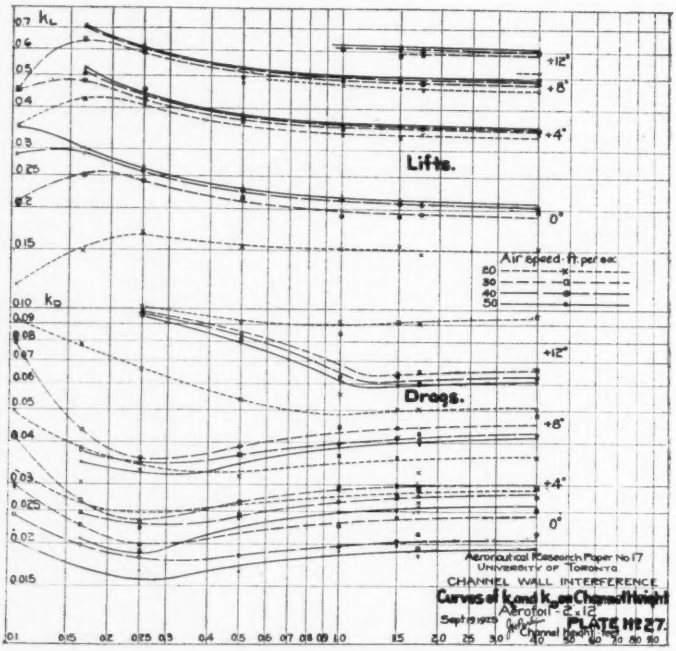
The left group of plottings on each plate show the pressures as taken at the end hole, that is the distribution of pressure around the cylinder 1.26in. from its end, with the end spindle removed.





The right set of curves on each plate shows the corresponding pressures, but with the 5/16in. dia. end spindle in place. A comparison of these two latter groups of curves shows the effect of the end spindle.

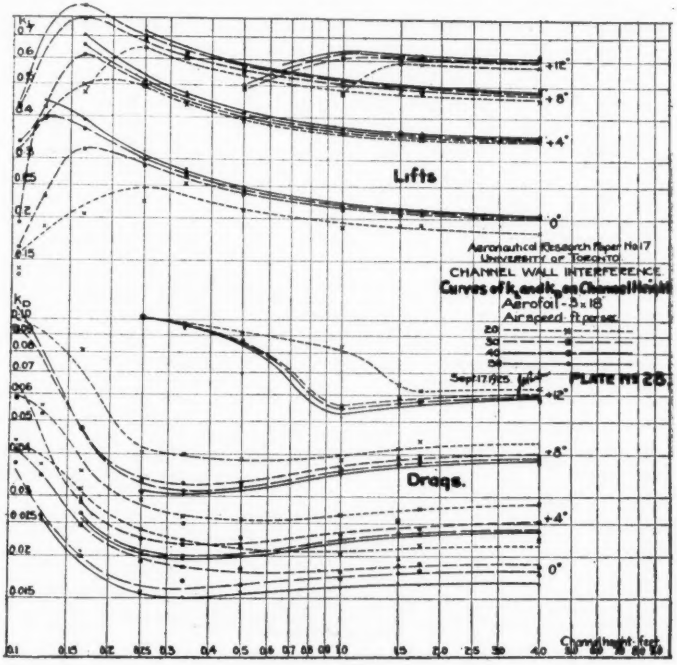
There are, in general, four curves in each of the above groups, each showing the distribution of pressure for a certain channel height. One curve shows the distribution for the full channel height of 4ft., or that is without mirror cylinders. The second curve shows the pressures for a channel height of about 0.333ft. The third curve shows the distribution around the cylinder for a channel height as close as practicable to the critical point or break in the curves of Plates 4-8. In the fourth curve the pressure distribution is shown for the minimum channel height of 0.166ft., equal to the cylinder diameter, where test and mirror cylinders are side by side and practically touching.



An examination of the curves showing the distribution of pressure around the median plane of the cylinder affords an explanation of the variation of drag with channel height. The full curve (central group Plate 14) for channel height equal to 4ft. is similar to that of Fig. 2, R. & M. 106, 1913-14, although the higher air speed of the present tests causes the distribution to be somewhat different.

As the channel height is decreased the curves show that the air banks up more and more on the front of the cylinders, the region of positive pressure becoming larger and larger, while at the same time the air jets between the test and mirror cylinders with higher and higher velocity, the pressure on the cylinder surface being correspondingly reduced. The point of minimum pressure, corresponding to the maximum jet velocity, moves farther and farther back in conjunction with the increase in frontal area of positive pressure. The higher the jet velocity, the lower the pressure in the dead or eddy region behind the cylinder. These effects all conduce to increase the cylinder drag as the channel height becomes less.

Beyond the critical point the air flow about the cylinders changes. There is a sudden decrease in the jet velocity between the cylinders, with the result that the suction on the sides and back of the cylinder is reduced. This is particularly noticeable for the central hole plottings on Plates 12, 13 and 14; the corresponding plottings on Plates 10 and 11 are in the transition region. The positive pressure area on the front of the cylinder continues to increase, but at a slower rate than the reduction of suction on the back, with the net result that the drag of the cylinder is considerably reduced. The three cylinders (two mirror and one test) are here acting partially as one body, in that the air is banking up in front on all three, but there is some leak through between the cylinders which serves to partially break the suction at the rear.



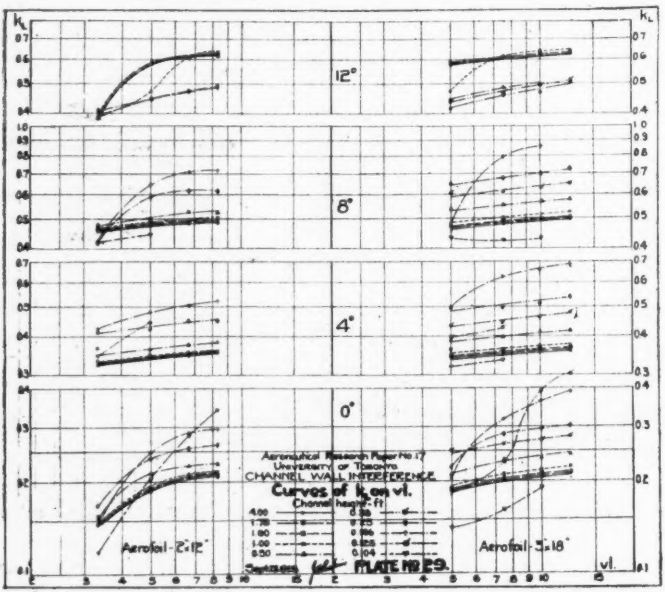
With further decrease in the channel height, down to the minimum where the cylinders are side by side, the air flow between the cylinders is more and more restricted until it finally practically ceases, the positive pressure exists over nearly the whole 180° of cylinder face, and at the same time the suction at the rear becomes larger, since it is now due to the dead region in the rear of three cylinders, practically unrelieved by any leakage between the cylinders. The three cylinders now function as a single body, the air piling up in front and leaving a single large dead region of low pressure in the rear. As a consequence, the drag of the test cylinder is here a maximum.

Comparing these curves of pressure distribution for the median plane for different channel widths, it is seen that the effect of decreasing the channel width is to decrease somewhat the region of positive pressure on the face of the cylinder and to increase the negative pressure on the back, the latter effect predominating so that there is a slight net increase in drag, as shown by the drag measurements.

The pressure distribution near the end of the cylinder is of the same general character as that at the centre, but the positive pressure region is smaller, while the negative pressure region and pressures are increased. This is doubtless due to the combination of normal air flow perpendicular to the cylinder axis with flow parallel to the axis set up by the passage of the air around the cylinder end. In other words, the air jets from the end as well as the two sides, and thus sets up a greater suction in the neighbourhood of the ends at the rear.

While for the central hole the suction at the back of the cylinder is always least for the channel height of 4ft., this is not the case for the end hole. The flow around the end of the cylinder is apparently much influenced by the proximity of the mirror cylinders (channel height). In most cases the minimum suction on the back of the cylinder at the ends occurs for a channel height of less than 4ft., due, it would seem, to end leakage. Nor does the maximum suction on the back occur for the minimum channel height of 0.166ft.

The presence of the 5/16in. diameter end spindle is seen to have very little influence on the distribution of pressure around the cylinder near the ends.



PART IV.

Tests on Aerofoils

In making the aerofoil studies it was not possible to arrange a guard of the aerofoil section around the standard of the balance and spindle, representing the end image (*i.e.*, the image in the floor of the channel). Hence, only the effect of variation in channel height was studied, the channel width remaining constant at 4ft.

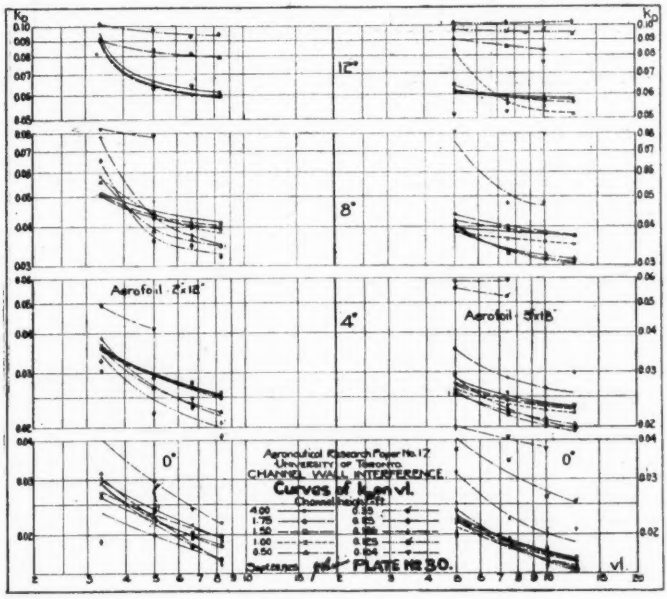
Three aerofoils were therefore used, that under test and two mirror aerofoils (see Figs. 5-7, Plate 2 and Photos 2-4). The 1¼in. guard tube around balance and spindle was duplicated, as shown, for the mirror aerofoils, and in deter-

mining spindle resistances dummy spindles and guards were arranged, supported from the rail in the roof of the channel.

Tests were made, with the 3in. × 18in. aerofoils, for ten channel heights, 4.00, 1.75, 1.50, 1.0, 0.5, 0.33, 0.25, 0.166, 0.125 and 0.104, with constant breadth of 4.0ft. For the 2in. × 12in. aerofoils tests were not made for the heights of 0.33 and 0.125ft. The minimum height of 0.104ft. is the smallest possible spacing, the $1\frac{1}{4}$in. diameter guard tubes being then in contact.

For each channel height the lift and drag* were measured at a series of incidences, −8, −4, 0, +4, 8, 12 and 16°. In certain cases measurements were made at intermediate angles where necessary to locate critical angles definitely.

Further, to study speed effects, each test was made at four air speeds, 20, 30, 40 and 50 feet per second.



Results

The observations, corrected for spindle interference, are tabulated as the usual non-dimensional lift and drag coefficients in Tables III.-VI., and have been plotted on curve sheets as follows:

Plates 15-22.—Lift and drag coefficients on incidence as base, one plate for each speed and model size. Because of the fact that the points were, in many cases, very close together, the actual observation points have not been plotted on these plates.

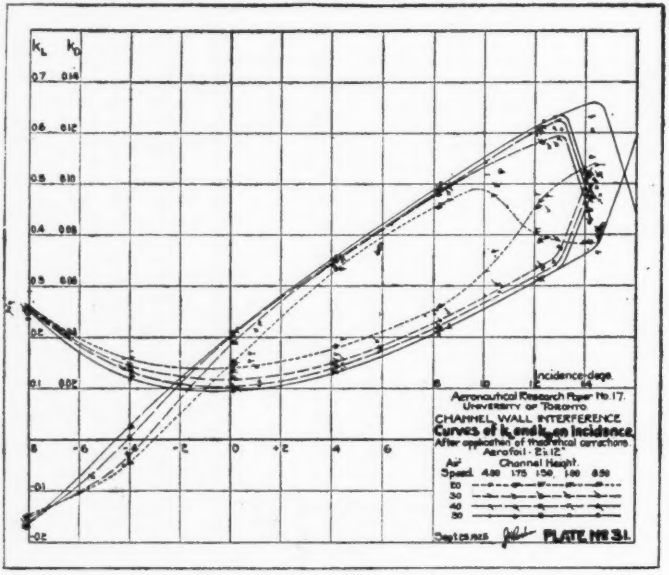
* The aerofoil here used is of the same section as that used previously for a research on thick aerofoils (see Aeronautical Research Paper No. 8, September, 1921). The models in this earlier research were of wood and were tested in the old 4ft. N.P.L. type wind channel at Toronto. The models in the present research are of metal, and tested in the new R.A.E. type channel, some four years later, by entirely different observers. Yet the agreement between the coefficients given in the earlier report and those here given for similar conditions, 40 f.p.s. and 4 by 4 channel, will be found, on comparison, to be surprisingly good. This affords some indication of the accuracy of the models used and the reliability of the observations.

Plates 23-26.—Lift coefficient on drag coefficient (for the four larger channel sizes only) for different speeds and channel sizes.

Plates 27-28.—Lift and drag coefficients on channel height as base for different incidences.

Plates 29-30.—Lift and drag coefficient on *vl* base for different channel sizes.

The figures for the low speeds, 20 f.p.s., and particularly for the smaller models, are erratic due to the small absolute magnitude of the forces, and in some cases have not been plotted. At all speeds and for both series of models the figures in the neighbourhood of the critical angle and at negative angles are also uncertain, due to the unstable nature of the flow.



Discussion of Results

Plates 15-22.—An examination of these curves of lift and drag coefficient on incidence as base discloses a number of points of interest :

(*a*) As indicated by theory the incidence corresponding to a certain lift decreases as the channel height is reduced. The presence of the channel walls results in an induced velocity which is equivalent to an " upwash." The effect is seen to be small until a channel height of about 1ft. is reached.

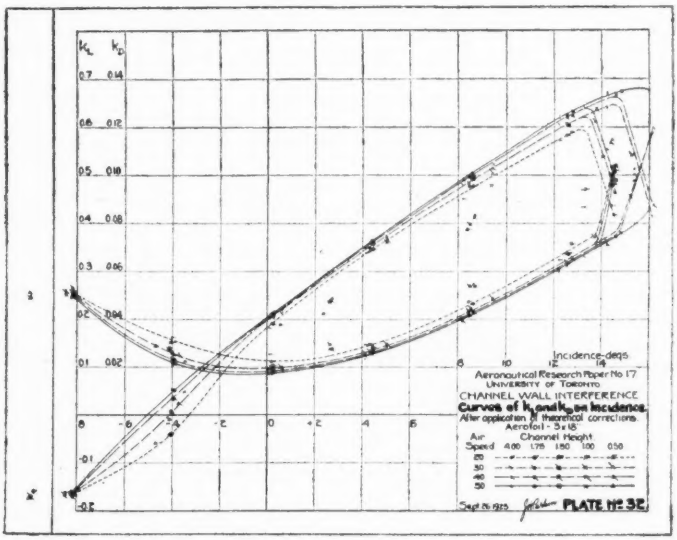
(*b*) There is a marked effect on the critical angle. Both angle and maximum lift are lowered with reduction in channel height until a height between 1.0 and 0.5 ft. is reached, beyond which, with further reduction in channel size, the reduction in angle is slight and the maximum lift exhibits a rapid increase until instability of flow is set up.

These results confirm the statement of McKinnon Wood (AERO. JOURNAL, December, 1922) that wall interference reduces the stalling angle.

It will be observed that for the very small channel heights the curves break off abruptly, sometimes at very low incidences. This was occasioned by the great

instability of flow occurring beyond these angles preventing the securing of further readings. The instability was so great as to cause the test aerofoil to vibrate to such an extent that, in certain cases, it came in contact with the adjacent mirror aerofoils, necessitating discontinuing the test. This instability varies with the speed occurring at a lower angle the greater the speed. Thus, for the 3×18 aerofoil and smallest channel instability occurs at $-4°$ at 50 f.p.s., and $+10°$ at 20 f.p.s., and for the 2×12 models at $0°$ and $+8°$ respectively for the same speeds. A similar effect is referred to in N.A.C.A. Tech. Note No. 67, p. 3.

As the channel height is decreased the induced velocity due to the channel walls (virtual upwash) increases, but at a slower rate than that of theory, until a peak or critical size is reached, beyond which there is an abrupt decrease. A breakdown or change in the type of flow apparently occurs at this point, as with the cylinders, as the space between the models becomes very restricted, with the


Incidence-degs
Aeronautical Research Paper No 17
UNIVERSITY OF TORONTO
CHANNEL WALL INTERFERENCE
Curves of k_L and k_D on Incidence.
After application of theoretical corrections.
Aerofoil - 3 x 18
Channel Height
Air Speed 4.00 1.78 1.60 1.00 0.50
PLATE No 3E

result that the lift is greatly decreased and the drag much increased. It would appear to be analogous to the burbbling action; in fact, it really means that the burbble point is brought so low that the flow over the whole test range, in certain cases, is of the type occurring beyond the critical angle.

With the 3×18 model at 20, 30 and 40 f.p.s. and for the minimum channel height, it was found that the flow was wholly of the kind existing beyond the critical, and it will be noted that the curves corresponding to these conditions differ materially from the others. The lifts are very low and the drags correspondingly high.

In certain cases, at high speeds, for small channel height and at negative incidences, the balance became very sluggish, giving the impression of some very viscous fluid surrounding the aerofoils.

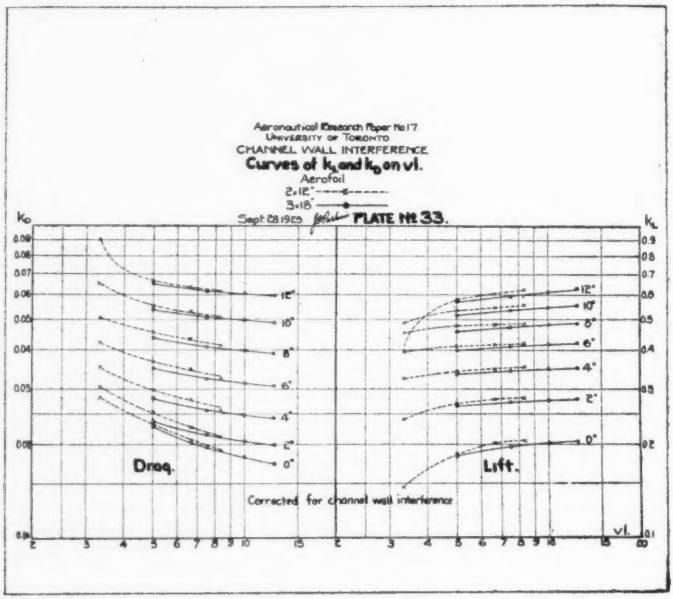
(c) The influence on the form of the lift curve at negative lifts is peculiar. With reduction in channel size the lift curve here develops a decided sag. This, it will be seen later, is partly a speed effect.

(*d*) According to theory the angle correction is proportional to the lift, and hence the lift curves should cross at zero lift. In no case does this occur for the experimental results. The curves, it will be observed, cross at a lift coefficient of about 0.05.

Plates 23-26.—These plottings were made to indicate the wall effects on the drag. It will be observed that the general effect is as predicted by theory, namely, the presence of the channel walls results in the drag being measured too low. The effect is seen to be small for the channel dimensions plotted.

Plates 27-28.—The lift and drag coefficients at several incidences and for the different speeds have been here plotted on the channel dimension as base, using logarithmic scales.

This plotting shows clearly the general effect of the channel walls on the aerofoil characteristics.



With reduction in channel size the lift increases slowly at first, then more rapidly, reaching a peak for a ratio of chord to channel height between 10/7 and 10/9, after which there is an abrupt and marked reduction in lift to values, in general lower for the minimum size of channel than for the full 4ft. channel.

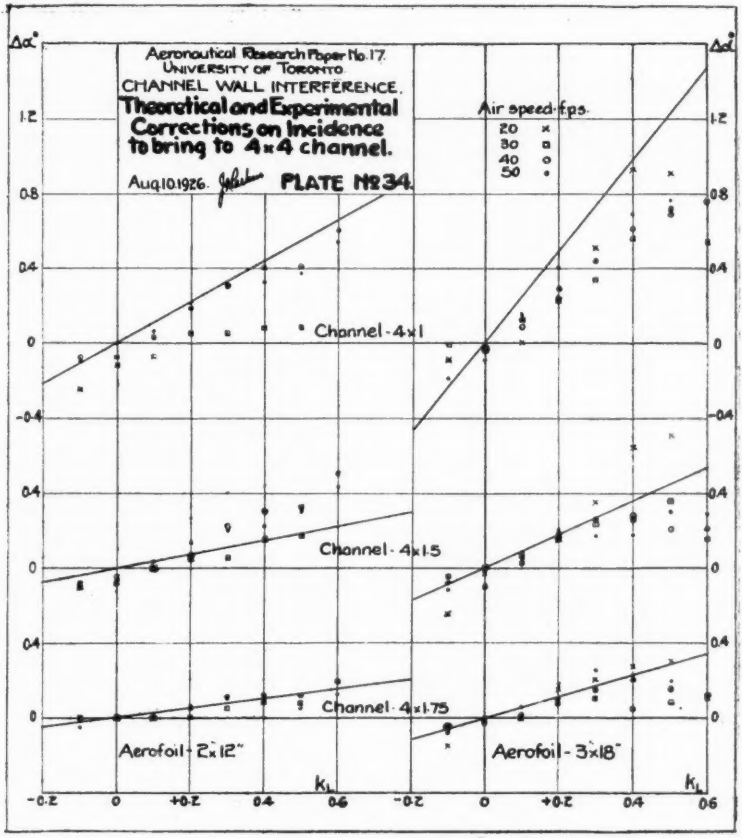
The drag, on the other hand, decreases with reduction in channel height, slowly at first, then more rapidly until a minimum is reached at a chord : height ratio of about two-thirds, beyond which there is a large and rapid increase in drag to values much in excess of those for the 4ft. channel.

This effect, it will be observed, is quite the opposite of that observed for the drag of cylinders, the latter increasing with channel height reduction.

The above statements apply to the higher *vl*'s only, as at lower *vl*'s and near the critical angle the critical points in these curves occur at larger channel heights. This corresponds to the observed *vl* effect on the critical point in the cylinder drag curves.

If the two Plates 27 and 28 are superimposed so that the 1.5ft. channel height for the 3 × 18 aerofoil (chord : height = 1/6) coincides with the 1.0ft. channel height for the 2 × 12 aerofoil (chord : height = 1/6) the agreement between the two sets of curves will be made very clear. The curves of the 2 × 12 model will then appear as the continuation of those for the 3 × 18 and the critical points will also appear in good agreement.

These curves show clearly that the reduction in interference to be secured by increasing the channel height beyond 4ft. for the 3 × 18 aerofoil can be but little or, on the other hand, that a channel of smaller height (say 2ft.) may be

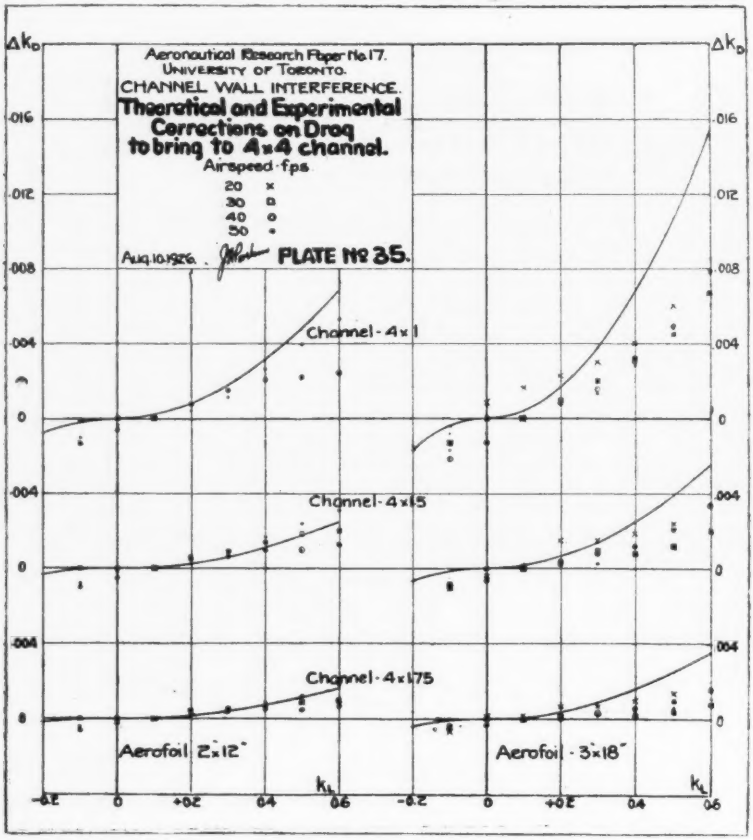


used without seriously increasing the interference effects. Or again, larger models, say 6 × 36, can be successfully used in the 4 × 4 channels. This practice has already been followed at Göttingen (refer McKinnon Wood, Int. Air Congress, London, 1923, p. 341).

Plates 29-30.—These plates contain plottings of the observed coefficients at various angles, on a $\sqrt{d}$ base, for different channel heights. The curves thus show the combined effect of channel wall and scale or velocity.

Plates 31-32.—The observed coefficients for the 4 × 4, 4 × 1¾, 4 × 1½, 4 × 1 and 4 × ½ channels were corrected to free air conditions, using the Prandtl cor-

rections, and are plotted on these plates. The curves drawn are average curves for each group of points for any one speed. It will be noted that the theoretical corrections secure satisfactory agreement of the different observations for channel heights as low as 1.0ft. for the 2 × 12 and 1.5ft. for the 3 × 18 models, that is a ratio of c/h = 1/6. Points for channel heights lower than these values were neglected in drawing the curves. The theoretical corrections are seen to be non-applicable for the 4 × 0.5 channel. Beyond the above limits, theory yields a definite " over " correction since the sequence of plotted point for lift progresses to the right and on drag upward and to the right.

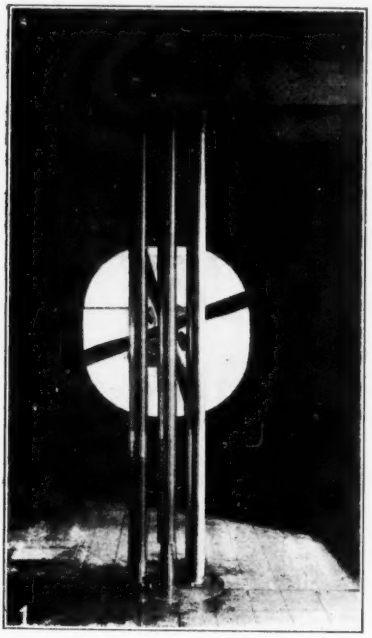


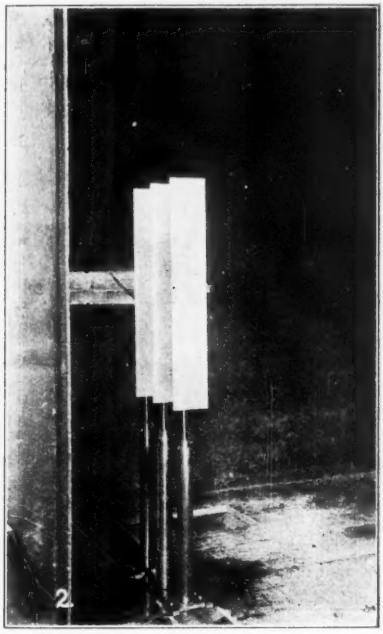
These curves show also the speed or so-called σl effect since spindle and wall interferences having been corrected, all other conditions except speed are identical throughout. Attention is directed to the effect on the lift curve at negative incidences, resulting in the characteristic sagging, and to the reduction in critical angle with reduction in speed.

Plate 33.—The curves on this plate have been plotted using figures taken from Plates 31 and 32, and hence show the scale effect on lift and drag coefficient. The curves are of the usual form. The agreement between the results for the

2×12 and 3×18 aerofoils is good. The fact that the coefficients for the 2×12 model are everywhere higher than those for the 3×18, while possibly due to experimental errors, may indicate the presence of another factor, as for instance roughness of model surface. While both models were finished with the same polish, that on the 3×18 would be relatively finer than that on the 2×12.

Plates 34-35.—These plates show graphically the agreement between theoretical and experimental channel wall interference corrections. The theoretical corrections to bring the results of tests on the 4×4, $4 \times 1\frac{3}{4}$, $4 \times 1\frac{1}{2}$ and 4×1 channels to free air conditions were first calculated and by subtraction the corrections necessary to bring the results of the smaller channels to those of the 4×4 channel were found. The theoretical quantities used in the computations are given in Table VII.





From Plates 15-22 the angle corrections necessary to bring the results to those for the 4×4 channel were measured, and similarly from Plates 23-26 the corresponding drag corrections were scaled.

The theoretical and experimental angle corrections are plotted on Plate 34, the full curve representing the theoretical corrections and the plotted points the experimental corrections. It will be seen that the agreement between the theoretical and experimental corrections is good for the 3×18 aerofoil for channels as small as $4 \times 1\frac{1}{2}$ and for the 2×18 aerofoils practically to channels 4×1.

On the other hand, the drag corrections similarly plotted on Plate 35 are not in as good agreement. The lack of agreement here is probably due to the very small absolute magnitude of the quantities involved together with the method of determining them, namely by subtraction. The agreement, however, is fairly good for lift coefficients between zero and 0.3. Glauert states (see R. & M. 723, p. 20) that for lift coefficients below 0.3—" the difference (between results in 4

and 7ft. channels) might be too small to determine reliably, and at higher lift coefficients the drag tends to become unsteady, but experimental confirmation of the theory might be obtained over the central region."

General Conclusions

1. The influence of the wind channel walls on the wind forces exerted on a symmetrical body, such as a cylinder or strut (and presumably on a streamline form such as a dirigible envelope), is to increase the drag. The drag will be read higher than the free air drag. The increase is slight provided the model is small relatively to the channel, but becomes appreciable for large models in relatively small channels.

2. The use of excessively large models in small channels is accompanied by instability of air flow in the channel, rendering measurements difficult and results unreliable.
3. The flow of air through a restricted space may be one of two kinds, depending apparently on shape and size of the space, curvature of walls and air speed. As the space is reduced the air originally flowing through at high speed is suddenly retarded.
4. The channel wall effects on the aerofoil characteristics are found to be as predicted by theory, for the closed channel the incidence and drag for a given lift are both measured too low.
5. The experimental corrections for wall interference, on angle and drag, are found to be in reasonable agreement with those predicted by theory for channels of fair size relatively to the model.
6. The channel walls reduce the critical or stalling angle of aerofoils.

7. The results demonstrate the possibility of using relatively much larger aerofoil models in the existing wind channels than is done at present without introducing excessive wall corrections. This will in turn permit greater accuracy in the measurement of the greater forces exerted on the larger models. The investigations show the extent to which this practice may be carried.

8. The behaviour of an aerofoil in an excessively restricted stream is shown, and the magnitude of the boundary corrections determined for channel sizes beyond the limits of applicability of theory.

List of Symbols

L = measured lift. K_L lift coefficient $= (L/\rho S V^2)$
D = measured total drag. K_D drag coefficient $= (D/\rho S V^2)$
D_o = profile drag. K_{Do} profile drag coefficient.
D_1 = induced drag due to finite span. K_{D1} induced drag coefficient.
D^1 = induced drag due to channel walls. $K_D{}^1$ coefficient.
c = wing chord.
s = semi-span of wing.
S = wing area.
A = aspect ratio of wing $= (2s/c) = (4s^2/S)$
R = radius of air stream or channel.
C = cross sectional area of air stream or channel.
h = channel dimension perpendicular to wing span.
b = channel dimension parallel to wing span.
V = air speed.
w = induced velocity—unlimited stream.
w_1 = induced velocity due to stream boundaries.
a = measured angle of incidence—radians.
a_o = effective angle of incidence $= a - w/V$.

A NEW TYPE OF COMBINED AIRSCREW HUB DYNAMOMETER AND THRUST METER

BY SQUADRON LEADER F. E. HELLYER, M.A., D.I.C., A.F.R.AE.S.

Introductory

From time to time attempts have been made to produce a dynamometer capable of measuring the power output of an aero-engine during flight at various altitudes. The type that has proved the most suitable up to date in flight tests is the Bendemann hub acting on the hydraulic principle which was originally designed in Germany. The principle has recently been tested in America with moderately satisfactory results, and experiments with a modified form of the apparatus are at present being conducted at the Royal Aircraft Establishment. The Bendemann hub and its accessories are somewhat complicated and are difficult to instal in a machine and to maintain in a serviceable condition, also the apparatus will not work properly at altitudes above 20,000 feet owing to increased viscosity of the working fluid at low temperatures.*

The apparatus described below is simpler in construction, lighter and more robust than the Bendemann hub, and should be capable of giving equally good results irrespective of altitude; moreover, the work of fitting it to a Service machine would not be great and would not necessitate the employment of specially skilled mechanics.

This instrument is designed to measure the resultant force upon its hub of the forces due to the airscrew's torque and thrust. The resultant being found, it is then easy to resolve it into its two components, the force due to torque (from which the engine power can be determined), and the actual thrust of the airscrew along its shaft. The accuracy of these results is unaffected by changes in density or air speed. A model, working up to six horse-power, has recently been constructed at the Air Ministry Laboratory, and, checked by a dynamometer, has obtained the brake horse-power of the motor which drove it, at various speeds, correct to within 1 per cent.

Description of the Apparatus and of the Principle Employed—General

The airscrew is mounted loosely upon an extension to the driving shaft in the manner shown in Fig. 1, balls being interposed between the bearing surfaces to reduce the friction. Under these circumstances, the propeller is free to rotate round the shaft or to slide along it. The drive of the engine is imparted to the airscrew through the medium of two inclined keys (6, Fig. 1) fixed rigidly to the driving shaft extension, and two rollers (5) one being mounted at the end of each of the two lugs (4), the other ends of the latter being fixed to the propeller boss (2). With the airscrew mounted thus, its thrust will cause it to move forward along its shaft, while, owing to the inclination of the driving key, the torque will tend to cause it to screw back along the shaft towards the engine.

If now the angle of inclination of the driving key were specially adjusted for the purpose, it would be possible to arrange for the tendency of the propeller to move forward exactly to balance its tendency to move back.

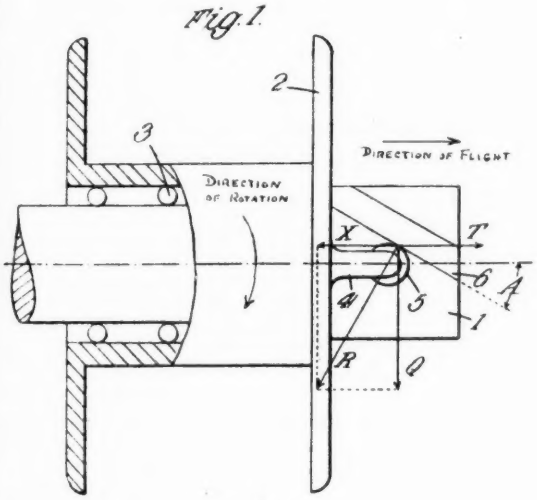
* See National Advisory Committee for Aeronautics Report, No. 252 of 1927, p. 10.

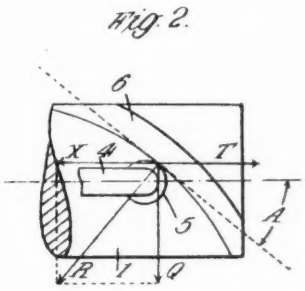
Measurement of Airscrew Efficiency and Engine Horse-power

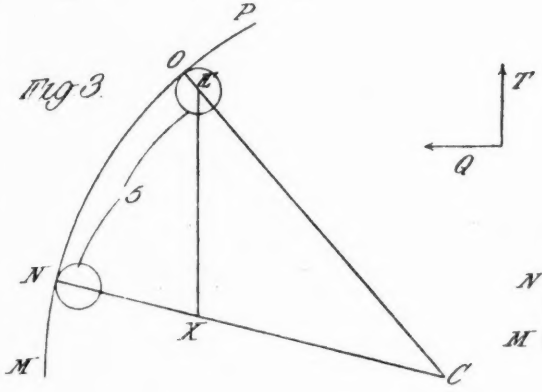
The efficiency of a propeller may be defined as

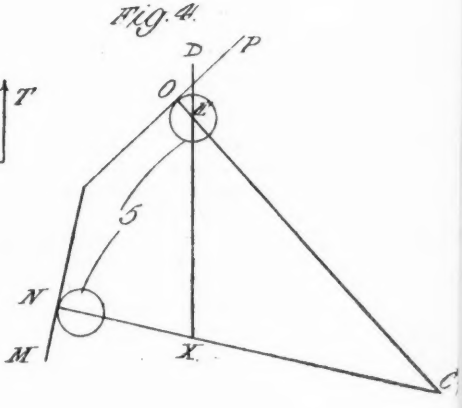
$$\frac{\text{Its thrust} \times \text{its forward speed}}{\text{Torque required to produce the thrust} \times \text{engine speed}} \times \text{a constant.}$$

This instrument (when used merely for propeller testing) is designed to give the ratio thrust/torque, and the method by which the ratio is obtained is either to adjust, during the test, the angle of the inclined key so that the airscrew is in a position of equilibrium along its shaft, and then to measure the angle, or









to vary slightly, by an internal force, the thrust of the propeller so that it "floats" in a position of equilibrium along its shaft and then to measure the external force required for the purpose.

Fig. 1 denotes the keyway (6) inclined at an angle "A" to the propeller axis. The line of flight and direction of rotation are indicated by arrows.

Supposing

T to represent the actual thrust of the airscrew.

R the total force exerted by the driving face (6) upon the roller (5) in turning the propeller.

Q to represent the component of R in the direction of rotation of the airscrew (*i.e.*, the force required to rotate it against its torque).

N to represent the component of R in the direction opposite to the thrust (*i.e.*, the force with which, owing to the inclination of the driving faces, the airscrew is twisted back towards its engine).

Under the influence of the above forces the airscrew will

(1) Be rotated by the engine at such a speed that its drag is exactly balanced by Q.

(2) Move along its own shaft in the direction of the greater of the two forces N and T, unless N and T are equal, or unless an external force is applied to balance the difference between N and T, in which case the airscrew will " float " upon its shaft while doing work.

Suppose the angle A to be such that N is equal to T, the resultant of the forces due to torque and thrust will be normal to the contact face of the driving key and the angle A will be such that $\tan A = T/Q$. From this value, provided the engine and air speed have at the time been noted, the propeller efficiency ($TV/QN \times$ a constant) can readily be determined. In view, however of the fact that the value T/Q must necessarily vary with different propellers or, using the same propeller, with different values of V/ND, a modification to the single straight driving face is necessary in practice for obtaining either the ratio of these two quantities or their actual values. These may be obtained in either of the following two ways, the latter being considered the more practical of the two.

(1) For the airscrew to be able to turn and slide on its shaft to the necessary extent while the roller moves along the surface of the driving key with its face in contact with it, it is necessary for the key to take the form, not of a flat inclined plane as shown, but of the side of a square sectioned screw thread of a pitch depending upon the desired angle of slope. Suppose now (keeping the diagrams of the sloping key two dimensional for the sake of clearness) " 6 " instead of being straight as in Fig. 1 is curved as in Fig. 2, the contact surface being, in reality, that of the side of a square sectioned screw thread of varying pitch. The propeller will then, under working conditions, slide forward to such a point along the key that the line of action of the forces due to torque and thrust is normal to the curve at that point. If now it be known what angle the tangent of the curve makes to the propeller axis at various points along its length, it is only necessary to observe the distance the airscrew hub has slid along its shaft from a previously determined " zero " to its position when running, to be able immediately to ascertain the value T/Q and hence to calculate the propeller efficiency.

Consider the diagram shown in Fig. 3. Here the roller " 5 " is shown at " O " in its forward " floating " position, the resultant of the forces due to torque and thrust, as explained above, being normal to the curve MNP at that point. The direction of the resultant is therefore determined as that of the normal OC, but its magnitude is not yet known. If the propeller now be pulled back along its shaft by any known force " P " acting in direct opposition to the thrust (the method of applying this force will be described later), the roller " 5 " will take up another position such that the line of action of the

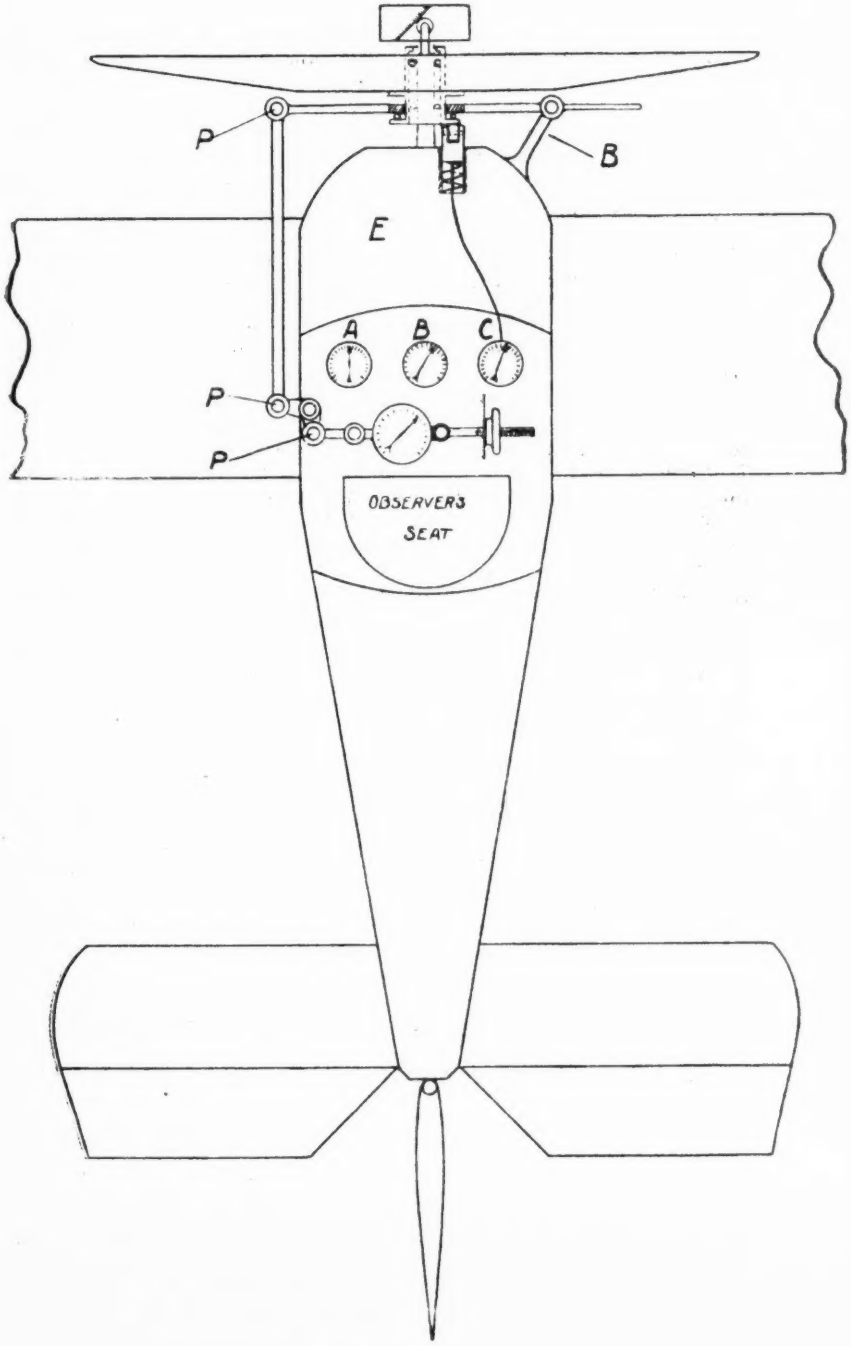


FIG 5.

resultant of the forces due to torque, thrust, and the force " P " is normal to the curve at the point where the roller is in contact with it; let this position be indicated by the point " N " in the diagram.

Draw normals from O and N and let them intersect as shown at C. Let X denote the point where the line of action of " P " cuts $N\ C$, also let E denote the centre of the axle of the roller when the latter is in its first position.

Then EXC may be taken to represent a triangle of forces to a scale such that EX represents " P " units of force, *i.e.*, that one unit of length represents P/EX units of force (regardless of the shape of the curve). In this triangle of forces, the values of EX and of the three angles (the latter from the angles made by the normals to the curve) are known, and the value of CE the resultant of the thrust and torque forces can therefore be ascertained. By resolving CE along and at right angles to the direction of flight, the actual thrust of the propeller and the torque of the engine at any moment can be determined.

It will be seen that by means of an instrument to indicate the position of the propeller along its shaft, the airscrew efficiency value T/Q can be read directly from its dial, and the engine power output can be obtained whenever required by observing the second position of the propeller, and then reading off the power from a calibration table previously prepared. It is considered that the above arrangement possesses the following advantages.

(*a*) Too long a traverse of the propeller hub along its shaft would probably be necessary for accurate readings.

(*b*) The apparatus to indicate the position of the airscrew hub relative to its shaft at any moment renders the apparatus complicated.

(*c*) The machining of helical driving faces of varying pitch would be difficult.

The reasons for describing the above form which the apparatus might take were :-

(*a*) Because the principle is novel and a useful application might be found for it.

(*b*) Because it might be adapted for bench tests of engines or airscrews.

(2) The following method of obtaining the values of the forces T and Q, which embodies the same principle in a more practical form, is considered much more promising and has been applied successfully to the model.

The lay-out is shown diagrammatically in Fig. 4. It will be seen that in each case two helices of known constant pitch are coupled in gang. The accurate machining of such driving faces presents no difficulty.

Now if the pitch of the more oblique of the two helices (*i.e.*, OP) be such that the tangent of its angle of obliquity to the airscrew shaft axis is less than the least value of T/Q it is desired to measure, the roller " 5 " under the action of torque and thrust would first take up a position at the forward end of the helix OP (say up against a front stop situated at P, not shown in drawing). On application of an external force P_1 in opposition to that due to thrust it could be made to " float " upon the first helix and to take up the position O as shown. On increasing the external pull by the amount P_2, the roller could be drawn back and made to " float " upon the second driving face MN and to take up any position N along its length.

Draw perpendiculars to OP and MN from O and N and let them intersect at C. From E draw EX parallel to the propeller shaft axis; let it represent P_2 in magnitude and direction. Produce XE to D; let DE be of such a length that it represents P_1 in magnitude to the same scale that EX represents P_2. Then as in the previous case EXC represents a triangle of forces to such a scale that one unit of length represents P_2/EX units of force. This triangle can be solved since EX and the angles are known and either CE or CX can be resolved along and at right angles to the propeller shaft axis to obtain the values of the remainder of the thrust (the fraction $P_1 + P_2$ being already known) and the torque force respectively.

This method possesses the following advantages:—

(1) The machining of the driving faces presents no difficulty.

(2) No instrument (other than that for applying the forces "P") is required to indicate the position of the airscrew along its shaft.

(3) A comparatively short travel of the propeller along its driving shaft will suffice for accurate measurements. A total travel of one inch should be ample for the purpose.

(4) The instrument requires no calibration, for provided the driving faces are accurately machined, the values it is desired to obtain may be determined from the following simple formulæ (these will be explained fully later).

$$\text{Brake horse-power} = N \times P_2 \times \text{a constant.} \qquad (N = \text{engine speed.})$$

$$\text{Airscrew thrust} = P_1 + P_2 \times \text{a constant.}$$

$$\text{Airscrew efficiency} = \frac{P_1 + P_2 \times \text{a constant}}{P_2 \times \text{a constant.}} \times V/N. \qquad (V = \text{air speed.})$$

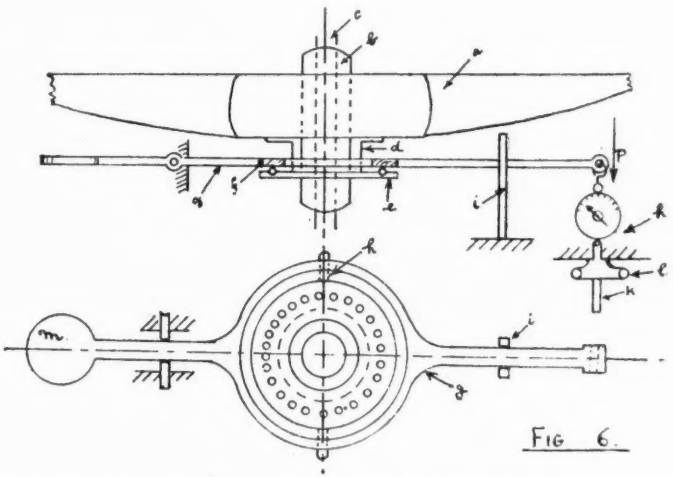
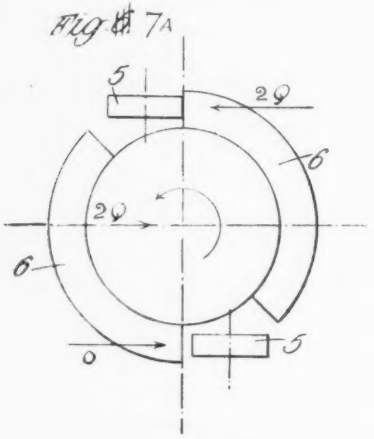


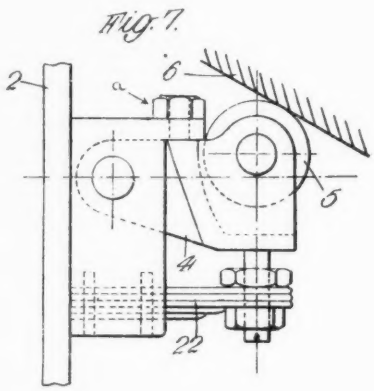
FIG 6.

Method of Applying the Force "P" Referred to Above

The lay-out of this part of the apparatus is shown in Fig. 6. Here "c" represents the propeller shaft and "b" the sleeve upon which the boss slides. To the end of the boss nearest the engine is fixed an annular bracket "d" with flanged ends; to the flange of this is fixed a thrust race "e," the attach-

ment being such that the thrust race runs concentric with the airscrew. A thrust bearing " f " of the usual type, which bears upon the balls of the thrust race, is located by two pivots " h " mounted at the top and bottom respectively of a circular opening in the arm " g." One end of the latter is held to a bracket from the aeroplane fuselage in such a way that the arm may swing fore and aft, but not up and down. A pair of guides " i," fixed to the fuselage, may be required properly to locate this arm and to steady it against vibration of the engine. A sail " m " may also be fitted, if necessary, to neutralise the effect of the wind upon the arm (neither the sail " m " nor the guides " i " were found necessary in the case of the model). One end of a spring balance " h " is connected to the arm, the other end being attached to a screw which can be worked backwards and forward by a nut " l " bearing upon a bracket fixed in a convenient position either outside or inside the aeroplane fuselage. By turning the hand nut " l," the force " P " is applied and measured by the spring balance.





Experiments with a Model

By the courtesy of the staff at the Imperial College of Science and of the Director of Aeronautical Research, facilities were afforded for the construction of a model designed for bench tests.

There were available, a motor producing about ten horse-power, and an airscrew of five feet diameter designed to consume this horse-power at an engine speed of approximately 1,600 r.p.m.

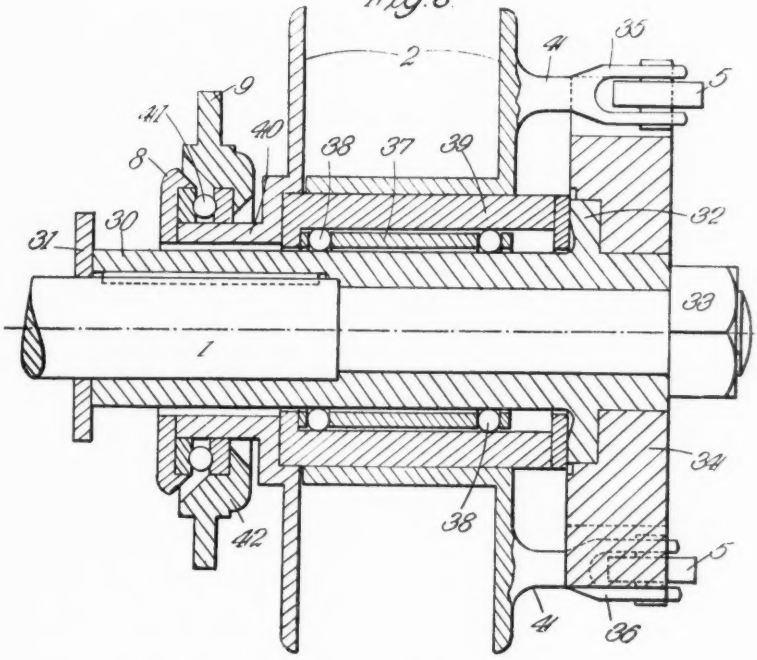
An outline of the general arrangement of the hub is given in Figs. 8 and 9. The following is a brief description of the chief components referring to the figures in the diagram :—

1. Engine driving shaft.

30. Extension fixed to " 1 " to carry the hub. It has two flanges on its outside circumference, one " 32 " solid with it and one " 31 " detachable. It is located on " 1 " by a key which holds it against torsion and by a nut which prevents it from moving forward under the action of the airscrew thrust.

34. Driving wheel. Fixed to " 30 " and concentric with it. It has two faces, " 35 " and " 36," which drive the propeller through the medium

Fig. 8.


2
41
35
5
9
41
40
8
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32
31
30
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1
34
42
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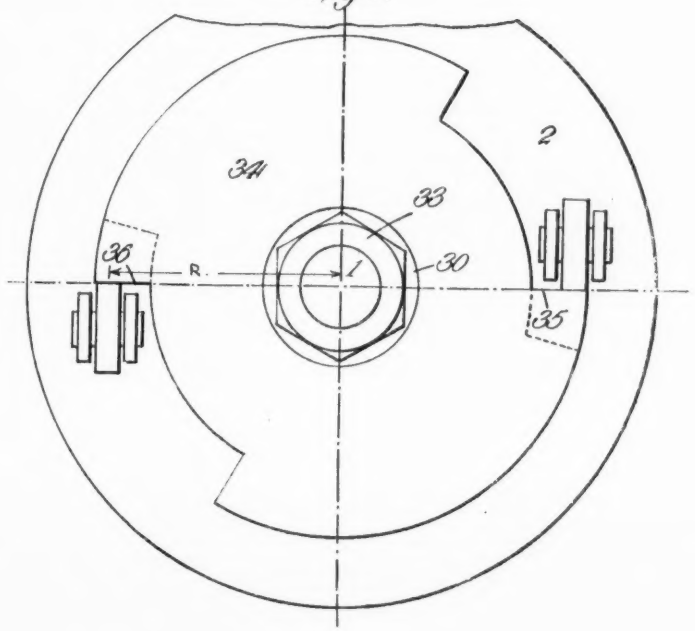
41
36

Fig. 9.


34
2
33
36
R.
1
30
35

of rollers " 5;" these faces are inclined at an angle to the axis of the shaft as explained above. The arms " 41 " holding the rollers " 5 " are pivoted to brackets on the forward propeller flange and are held in position by leaf springs, thus to ensure that the pressure of each of the two faces " 35 " upon its roller is of equal magnitude. Without such a device, unless the driving faces are machined with perfect accuracy, the reactions would in certain positions be unequal, and a varying load on the main bearing would result. Details of the pivotted and sprung roller are given in Fig. 7.

2. Propeller boss flanges (propeller fits in between them).

37, 38 & 39. A " floating " bearing in its housing capable of allowing the propeller hub either to rotate or to slide on " 30." -

Note—(*a*) When either sliding or rotating, the traverse of " 37 " is one-half that of " 39 " relative to " 30."

(*b*) The mechanism is here shown in its forward position.

8, 9, 40, 41 & 42. Thrust race mechanism for applying the external force " P " to the propeller.

Note.—The arm to which " 9 " is pivoted is not shown here; the arrangement, however, is shown in Fig. 6,

In the model, the traverse of the hub along " 30 " is limited to $\frac{5}{8}$ inch (half this movement would have been ample for the purpose), its movement being limited by the two stops " 32 " forward and " 31 " aft. The traverse of " 37 " is 5/16 inch; it has its rear and forward stop, the latter of which is detachable.

Two sets of the inclined driving faces " 35 " were made, the helices of each being of constant pitch. The mean radius " R " in Fig. 9 of the driving faces from the shaft axis was $3\frac{1}{2}$ inches and the pitch of the two sets of helices was 22 inches and 44 inches, making equivalent angles (in two dimensions), at the mean radius, of 45° and 26° 34′ respectively.

Friction

To reduce the friction as much as possible (1) the airscrew boss has been mounted on its shaft with a special ball bearing constructed so as to allow it either to slide or to rotate upon it. (2) The sliding friction between the inclined driving keys and the arms which transmit the torque is minimised by the interposition of ball-bearing rollers and (3) The force " P " is applied through the medium of a ball-bearing thrust race.

In view of the desirability of thoroughly understanding and reducing, as far as possible, any lag due to friction that may occur in such an apparatus, the friction which may arise between each of the above three moving parts will be dealt with separately and at some length.

Friction Between the Propeller Boss and the Shaft on which it Slides

It is reasonable to assume that the coefficiency of friction of a ball bearing does not exceed .004. Suppose the propeller, boss, etc., to weigh 40 lbs.; then the frictional resistance to motion in this bearing, if no forces other than the weight act upon it, should not exceed .16 lb., which may be considered as negligible. If, however, one of the two rollers bears against the surface of its driving key much more heavily than the other, and consequently takes much more of the torque, the frictional force, estimated above, may be considerably exceeded. Suppose, for instance, that, owing to inaccurate machinery of the driving keys, one roller takes all the force due to torque, say 135 lbs.; then this force will appear also in the bearing and its frictional resistance will increase

by an amount $.004 \times 135$ lbs. $= .54$ lbs. (see Fig. 7a). When dealing with greater engine powers, this friction resulting from the force due to an unbalanced couple might assume considerable proportions. It is thought that in certain thrust meters that have been designed and tested on full scale, the omission to guard against the imposition on the main bearing of heavy and uncertain loads due to unbalanced torque forces has been one of the chief causes of failure.

To obviate the possibility of the above trouble occurring in this instrument, the arms " 41 " connecting the rollers " 5 " (Fig. 8) to the propeller boss have been pivoted; they are held in their normal position by leaf springs whose initial tension is adjustable. This component is illustrated in figure 7. Thus they can be made free to swing back if a pressure exceeding the normal force Q is applied to them. This ensures that, even if slight inaccuracy in the machining of the driving faces exists, the pressure on one key cannot exceed that upon the other by an appreciable extent, and that therefore the friction between the airscrew boss and its shaft cannot vary.

The component above described is thought to possess the following additional advantages.

(1) Through slight " give " in the springs, it smooths out rapid fluctuations in torque or thrust such as those due to the engine impulses or to the airscrew blades passing just in front of obstructions to the slip stream.

The effect of this and of the fact that no irregular extra load is thrown upon the main bearing will tend to lessen vibration, and wear of the bearings through " hammering."

(2) It has been found in the model that there is no tendency for the rollers to bounce off their driving faces, even if the motor is switched off when running at full speed. This enables slight backlash play to be allowed, and thus ensures that the only forces affecting the movement of the airscrew hub upon its shaft are those imposed by the driving keys upon the rollers. Backlash will only take place when the engine backfires on being started up or when the airscrew is " windmilling."

Note.—In this connection it is interesting to observe that by modifying the mechanism for producing the external force so that the latter may be applied in a forward direction as well as back, it would be possible, by using the backlash key (which would be a helix running parallel to the driving face), to measure the force exerted by the airscrew while " motoring " the engine at various altitudes and engine speeds, also the drag of the airscrew while doing so.

(3) The natural frequency of torsional vibration of the airscrew and its shaft is considerably lowered and by initial tensioning of the spring is adjustable within certain limits.

(4) By means of backward and forward adjustment of the normal position of the roller (by nut " a " in Fig. 7) correct alignment is ensured.

(5) A pair of pivoted and sprung driving arms as described above, but in a slightly modified form, could be used for economising power while driving certain machinery through the lessening of the friction in the bearings.

Friction Between the Driving Faces and the Rollers

The motion of an ordinary roller along the side of a square sectioned screw thread is a slipping, as well as a rolling one. This is made clear by reference to Fig. 10, from which it will be seen that, if the roller " B " is to rotate round the screw thread " S," at the point of contact at the outer edge of the roller, slipping must take place in the direction of motion of the latter relative to the thread, and at the inner edge, the same amount of slip will arise, but in the opposite direction.

One method of overcoming this friction would be to use a roller with bevelled sides, and a screw thread with its side similarly inclined, the bevel being such that $d/D=r/R$ (see figures 11 and 12).

This expedient was not found necessary for the proper working model.

Friction in the axle of the roller was minimised by using ball bearings.

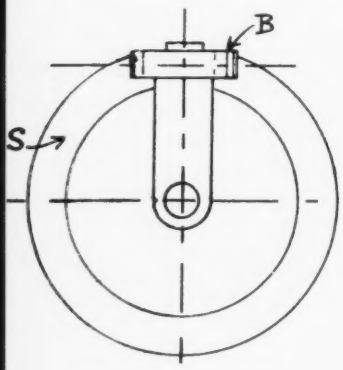


FIG 10.

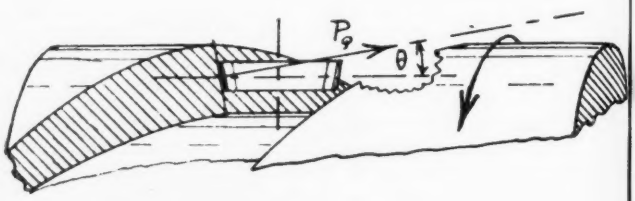


FIG 12.

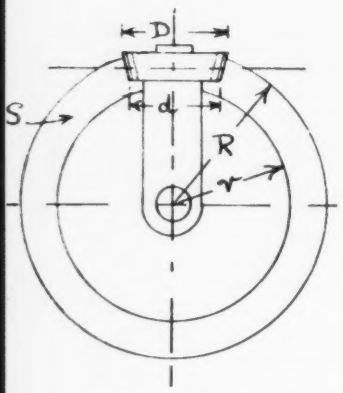


FIG 11.

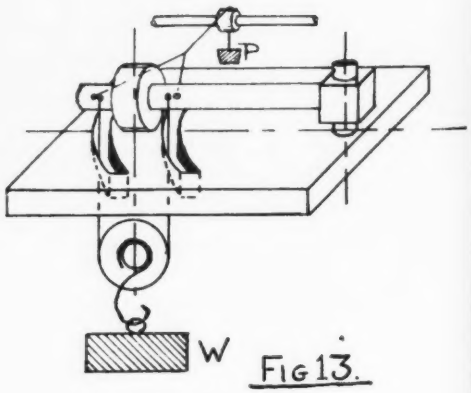


FIG 13.

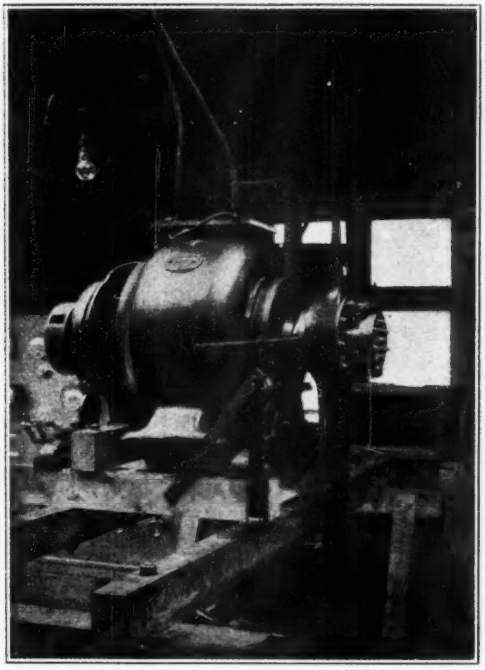
The apparatus illustrated in Fig. 13 was used to determine what would be the coefficient of friction of the roller mounted as shown. The roller was loaded by various weights " W " and the force " P " required to set it in motion was measured. The roller was then disconnected from the arm, and the experiment repeated, the motion of the roller upon the flat surface being, in this case, purely a rolling one.

The coefficient of friction for the circular motion was found to be .0116, and for the straight line motion .0034. This friction would in theory induce

an impedence to the motion of the apparatus of just under 1 per cent. of the total airscrew thrust, but in practice, owing to vibration, it is found that friction is practically non-existent.

Full Scale Work—Weight

The weight of the complete model (less that of the airscrew) is 35 lbs. (approx.); in its design, however, no attempt was made to keep the working parts light. These were purposely made solid to ensure ample strength, castings being used for economy, instead of forgings, and no light alloys such as aluminium being employed. The mean radius of the driving faces (measured



from the shaft axis to middle of the rollers) was $3\frac{1}{2}$ inches. It is estimated that the apparatus, if properly designed, for full scale work for use with engines up to 250 h.p., would weigh approximately 90 lbs. (not counting the weight of the airscrew). This approximation is arrived at on the assumption that the mean radius of the driving faces would be 7 inches. Under these circumstances, the pressure between each driving face and its roller would be in the neighbourhood of 700 lbs. This load could well be taken by a $2\frac{1}{2}$in. or a 3in. Skefko Standard Type single-row radial bearing; the former size weighs under $\frac{1}{2}$ lb. and will take a steady load, when moving slowly, of 1,400 lbs. A self-aligning bearing should not be used for this purpose.

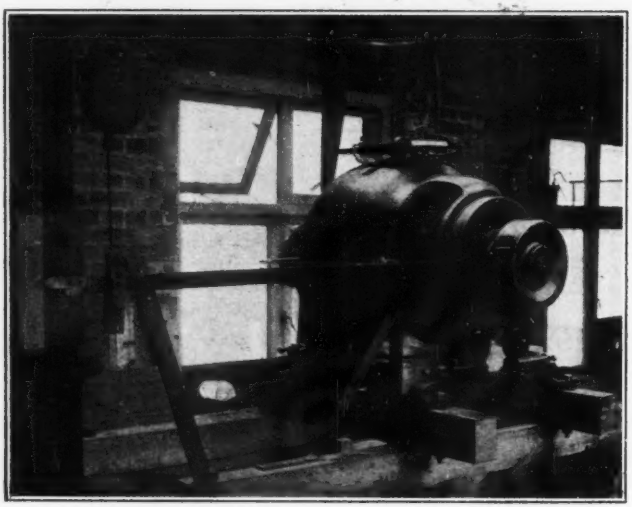
In the central bearing, either 2 or 4 rows of $\frac{3}{8}$in. or 7/16in. balls should suffice. In the model, 2 rows of 5/16in. balls have been used in the central bearing, and two 30mm. diameter single radial bearings as the rollers; after

about 20 hours running, the apparatus was dismantled and inspected, no sign of wear being found.

The thrust race, through which the force " P " is applied, is very lightly loaded and the lightest bearing of the requisite diameter procurable can be used in this position.

General Observations

A suggested lay-out for the apparatus for use in an aeroplane is shown in Fig. 5, the diagram being self-explanatory. *A* and *B* represent the Air Speed indicator and Tachometer, *C* the apparatus for indicating the position of the airscrew along its driving shaft (this is not necessary if a pair of constant pitch driving faces coupled in gang are used).



The spring balance of measuring the force " P " is shown; it is suggested that it should be provided with a moveable dial (as is used in standard aneroids) to do away with the necessity of noting the " zero," or the force P_1 where this is not required (as in horse-power measurements).

There are other methods than that shown for applying the forces P_1 and P_2; an alternative would be to use the hydraulic principle; the pressure required would not be great and could be supplied from a hand pump. In this case a pressure gauge would be calibrated to indicate the " pull-back " force in lbs. Accuracy for readings is not affected by their being taken while the machine is climbing or diving, the only thing which alters being the spring balance " zero."

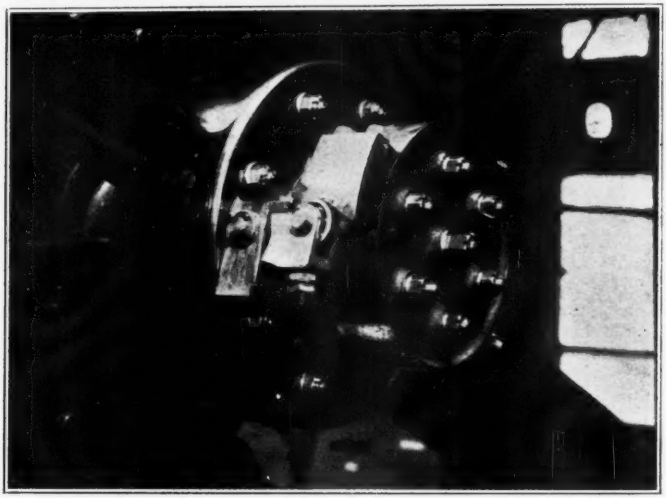
If the machine's line of flight undulates slightly while readings are being taken, the undulations, if not large enough materially to alter the engine speed, will not affect the accuracy of the instrument, because the acceleration due to gravity acts upon the whole system and not only upon the airscrew.

It may happen that, owing to slight fault in the machining or imperfect adjustment, the centre of pressure of the roller on the face is at a greater or less radius than that assumed. Such an effect does not influence the calculations, inasmuch as the pitch of the surface is the same at all radii, and an

error in the radius is exactly compensated by the corresponding error in the assumed angle of the surface to the axis.

Experiments with the Model Torque-Thrust Meter

When completed, the model was attached to the shaft of a 10 horse-power electric motor which was mounted on rollers and fitted with a dynamometer arm to check the instrument's measurements of horse-power. It was first put through a series of trials extending in all over about 20 hours' running, during which time the torque and thrust readings proved consistent and reliable. The apparatus was then dismantled, examined for wear and remounted for further trials with the second pair of driving faces. Since the idea of mounting two helical driving



faces in gang had not been conceived when the apparatus was designed, provision had only been made for mounting and testing them one at a time. Accordingly the external force required to "float" the airscrew along its shaft was measured at various engine speeds first with the faces having a pitch of 44 inches, this set of forces being called P_1, and then with those of 22-inch pitch, this set being called P_1. Since the experiments extended over some days, corrections were made to a standard air density, the standard chosen being Barometer 30 inches of mercury at 15°C., temperature 15°C. A number of these corrected observations of P_1 and P_2 were then plotted against their relative engine speeds, smooth curves being drawn through each set of readings. The brake horse-power values at various speeds were then calculated from the P_1 and P_2 curves in the manner explained in Appendix I. and plotted against engine speed. Dynamometer readings were taken every time a P_1 or P_2 observation was made. The apparatus was then taken off and the power losses in the motor's armature, bearings, etc., noted. The difference between the full horse-power registered and the motoring losses would give the power consumed by the airscrew. These differences accordingly were plotted against engine speed, a smoothed curve drawn through the observations and then the horse-power readings obtained with the torque-thrust meter as explained above were superimposed upon it.

A specimen of the resulting curves is shown in Appendix II., the experimental observations from which they were obtained being also given. It will be observed that the torque-thrust meter readings agree very closely with the average of those obtained with the dynamometer, details of divergencies between them and the smoothed curves and torque-thrust meter observations being as follows:—

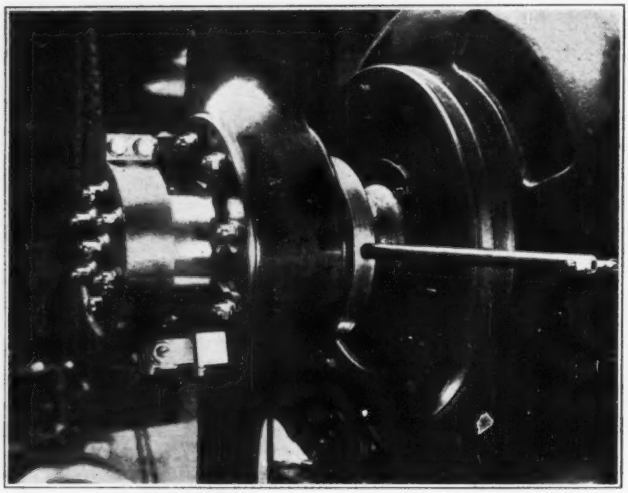
BRAKE HORSE-POWER.

Torque meter b.h.p. curve agrees with the average of the b.h.p. as recorded by the dynamometer to ... ... 1 per cent.

THE CURVES P_1 and P_2.

Average deviation of the experimental observations from the smooth curve is ... ... 1 per cent.

Maximum deviation of the readings from the smooth curve is ... ... 2 per cent.



No difficulty was experienced in observing the values of the forces P_1 and P_2 from the spring balance. It was found that the pointer of the latter, when the motor was rotating at a high speed, oscillated consistently about a well-defined mean value, the extent of the oscillations being about 3 ounces each way. The spring balance pointer occasionally gave a jump of about 7 ounces either above or below the mean, this being believed to be due to the harmonising of the different periods of vibration of the component parts of the connections between the apparatus and the spring balance. All the observations recorded in this report were made without the assistance of any artificial damping. An oil dash pot and piston have since been experimented with, and were found considerably to lessen the pointer's oscillation without altering the mean value; another method might be to use a spring balance having a different natural frequency. No difficulty is experienced in determining when the airscrew is in a "floating" position along its shaft, since, under these circumstances, several turns of the nut by which the force is applied to the spring balance produces no effect upon its reading, whereas, if the propeller is not "floating," any adjustment of the nut will cause the force registered by the balance to vary

considerably. Photographs of the model in position with a five-foot diameter propeller are shown at the end of the report.

My thanks are especially due to the Director of Scientific Research for permitting the erection and test of the apparatus to be conducted in the Air Ministry Laboratory, also to Professor L. Bairstow of the Royal College of Science, Captain R. O. King, Dr. H. Moss and Captain E. C. Horsley of the Air Ministry Laboratory, for most valuable assistance and advice.

Appendix I.

Derivation of formulæ for obtaining brake horse-power and thrust valves from the torque-thrust meter, using two constant pitch helical driving faces connected in gang.

See Figs. A1 and A2.

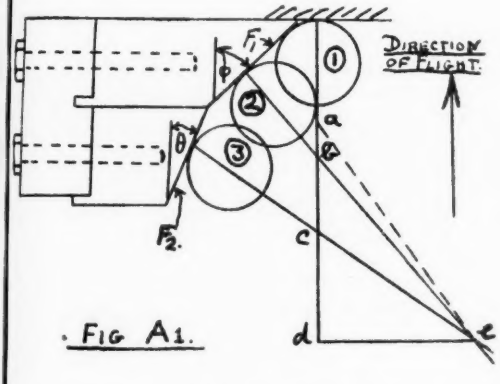


FIG A1.

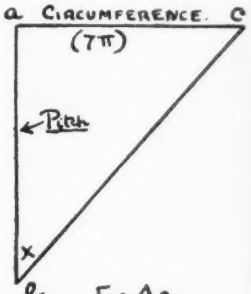


FIG A2.

Supposing F_1 and F_2 (in the figures) to represent two inclined driving faces (helices of constant pitch in reality, but shown here in two dimensions for the sake of clearness) making angles of ϕ and θ respectively with the line of axis of the propeller shaft. The most forward position of the airscrew along its bearing will be position (1). It can be pulled back, first, to an equilibrium position (2), and then to position (3) as shown in the figure.

Draw perpendiculars from any points along the length of F_1 and F_2; let them intersect at e and cut line of shaft axis at b and c respectively.

Draw de perpendicular to bc produced.

Let bc represent the force necessary to pull airscrew from (2) to (3). Set off on same scale (along db produced) ab to represent force to pull from position (1) to (2). From the construction, angle $bed=\phi$ and angle $ced=\theta$.

Then T total thrust is represented rectorially by da.

Q total torque force is represented rectorially by ed.

$T/Q=\tan aed$.

To Find the Brake Horse-Power

Any of the triangles in Fig. A may be considered as triangles of forces to the scale—1 unit of length = force to pull from (2) to (3)/length bc.

Now

$$bc/\sin(\phi-\theta)=be/\sin(90+\theta)=be/\cos\theta$$
$$\therefore\ be=bc\cos\theta/\sin(\phi-\theta)$$
$$Q=de=be\cos\phi=bc\cos\theta\cos\phi/\sin(\phi-\theta)=bc\times\text{a constant}$$

B.H.P. $=2\pi rNQ/33000$ where N = engine r.p.m. and r = radius of driving faces in feet.

$$=Nbc\,(2\pi r/33000)\cos\theta\cos\phi/\sin(\phi-\theta)$$
$$=Nbc\times\text{a constant.}$$

Note.—In the case of the model, B.H.P. $=Nbc/9000=N(P_2-P_1)/9000$.

The airscrew thrust

$$=ab+bd=ab+be\sin\phi$$
$$=ab+bc\cos\theta\sin\phi/\sin(\phi-\theta)=ab+bc\times\text{a constant.}$$

Note.—For the model, thrust $=ab+2bc$
$$=2P_2-P_1$$

Notes

(1) The above constants depend only upon the pitch of the helical driving faces. Once the apparatus is assembled the constants cannot vary unless the driving faces are changed.

(2) A certain percentage error in the measurement of ab and bc will cause a *corresponding* percentage error in the results.

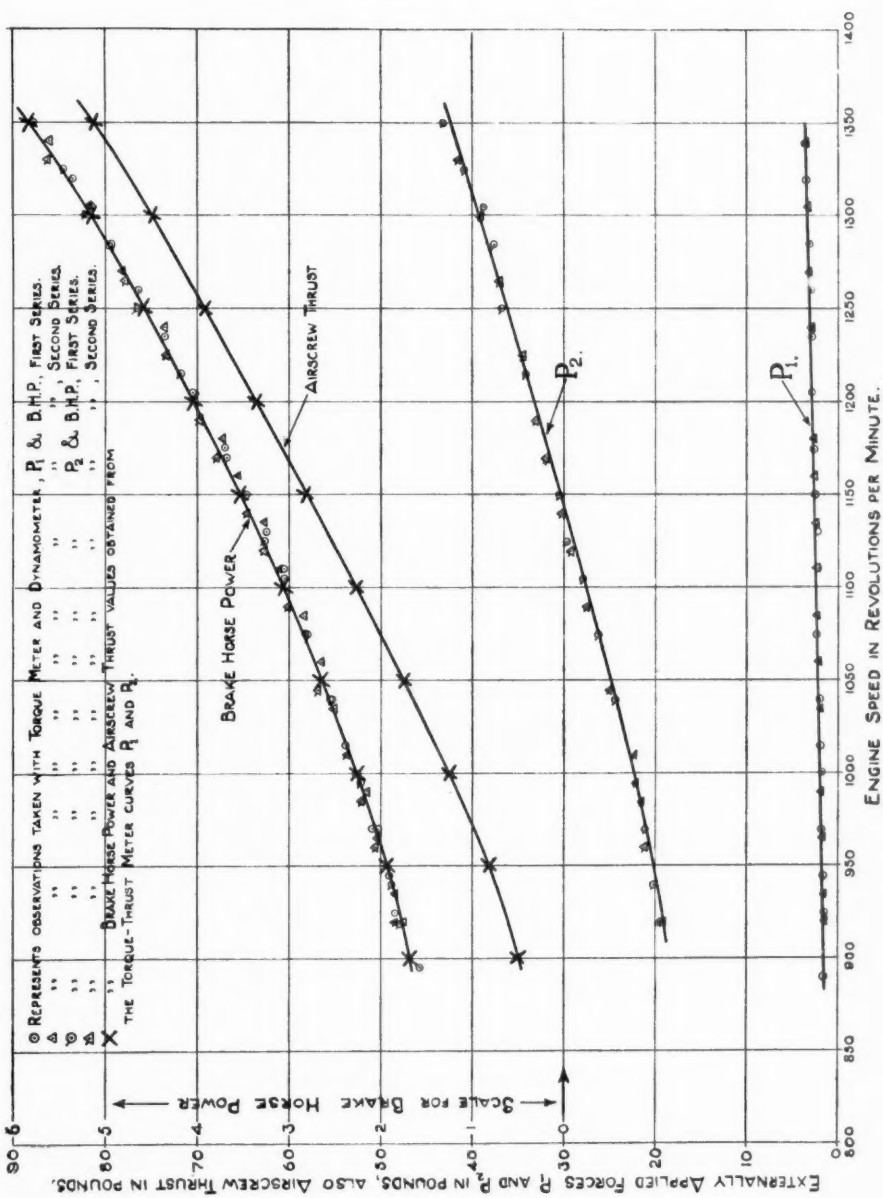
(3) The constants can, of course, be found or verified experimentally by using a dynamometer in conjunction with the apparatus.

Appendix II.

1.—Table of Observed Values of Engine Speed Power, P_1 and P_2.

Engine Speed, R.P.M.	Dynamometer, B.H.P.	P_1 lbs.	Engine Speed, R.P.M.	Dynamometer, B.H.P.	P_1 lbs.
890	1.58	1.50	920	1.76	1.61
925	1.84	1.57	935	1.85	1.61
945	1.90	1.57	965	2.04	1.74
970	2.09	1.74	990	2.16	1.80
1000	2.29	1.81	1035	2.51	1.98
1015	2.39	1.81	1060	2.66	2.17
1040	2.53	1.93	1085	2.84	2.36
1075	2.82	2.12	1110	3.09	2.42
1110	3.07	2.31	1135	3.28	2.47
1130	3.25	2.38	1160	3.57	2.53
1150	3.48	2.44	1180	3.73	2.73
1175	3.70	2.55	1240	4.37	2.91
1205	4.05	2.81	1270	4.82	3.04
1235	4.37	2.87	1305	5.18	3.22
1260	4.64	2.93	1340	5.61	3.41
1285	4.98	3.06			
1320	5.36	3.18			

Specimen of results obtained with Torque-Thrust Meter compared with those obtained with a Dynamometer.


Represents observations taken with Torque Meter and Dynamometer, P_1 & B.H.P., First Series.
" " " " " " " " Second Series.
" " " " " " P_2 & B.H.P., First Series.
" " " " " " " " Second Series.
Brake Horse Power and Airscrew Thrust values obtained from the Torque-Thrust Meter curves P_1 and P_2.
Brake Horse Power
Airscrew Thrust
P_2.
P_1.
Scale for Brake Horse Power
Externally Applied Forces P_1 and P_2 in pounds, also Airscrew Thrust in pounds.
Engine Speed in Revolutions per Minute.

Specimen of results obtained with Torque-Thrust Meter compared with those obtained with a Dynamometer.

Engine Speed, R.P.M.	Dynamometer, B.H.P.	P_2 lbs.	Engine Speed, R.P.M.	Dynamometer, B.H.P.	P_2 lbs.
920	1.78	19.23	920	1.85	19.30
940	1.88	20.10	960	2.06	21.20
970	2.03	21.10	985	2.22	21.70
995	2.21	22.10	1010	2.38	23.20
1040	2.54	24.30	1045	2.70	25.00
1075	2.81	26.10	1090	3.02	27.60
1105	3.06	27.90	1120	3.29	29.15
1125	3.27	29.60	1140	3.46	30.35
1150	3.48	30.40	1170	3.80	31.90
1170	3.69	31.80	1190	3.99	33.10
1215	4.19	34.10	1225	4.35	34.50
1250	4.67	36.75	1265	4.80	37.00
1285	4.94	37.65	1300	5.21	39.00
1305	5.18	38.65	1330	5.63	41.80
1325	5.45	40.80			
1350	5.78	43.00			

2.—Values of Brake Horse-Power and Airscrew Thrust obtained from Torque Thrust Meter Observations (*i.e.*, from Curves of P_1 and P_2).

Engine Speed, R.P.M.	P_2 lbs.	P_1 lbs.	$P_2—P_1$ lbs.	Brake Horse Power. From Torque Meter.	Brake Horse Power. From Dynamometer.	Airscrew Thrust, lbs.
900	18.3	1.5	16.8	1.68	1.68	35.1
950	20.0	1.7	18.3	1.93	1.94	38.3
1000	22.3	1.9	20.4	2.27	2.25	42.7
1050	24.8	2.1	22.7	2.65	2.63	47.5
1100	27.5	2.3	25.2	3.08	3.05	52.7
1150	30.4	2.5	27.9	3.55	3.52	58.2
1200	33.1	2.7	30.4	4.06	4.04	63.6
1250	36.0	3.0	33.0	4.59	4.59	69.2
1300	39.0	3.2	35.8	5.17	5.17	74.8
1350	42.4	3.5	38.9	5.84	5.80	81.3

REVIEWS

The Old Flying Days

By Major C. C. Turner. Sampson Low, 25/- net.

Major Turner is to be congratulated in giving us a book for which we have all been waiting—a book that perhaps some of us may have wished to write were it not for the magnitude of the research and industry necessary for such a task. In fact *The Old Flying Days* falls from the press already a standard authority in its first moments of life, for it is a book that no future historian or editor impatient of accuracy will be able to ignore.

In a hundred years time Major Turner will still be the standard authority, in two hundred he will be the grandson of Halton Turner, in three hundred he will be Halton Turner himself, such being fame.

But whatever happens to Major Turner, his book will remain, and to us, who followed aviation from its birth or joined the procession as it passed, it is full of enchanting memories of a glorious time. For a glorious time we had, in those far-off days before the war; so much so that even now the smell of burnt castor oil can fill us with poignant emotion.

And here they are all brought back by Major Turner's skill—the blue eyes of Charley Rolls, prescient with his early fate; the burly white-smocked figure of Cody shouting through a megaphone; Horace Short interrupting a diverting story to utter an amazingly acute prophecy of the performance of some untried machine; "Jimmy" Valentine, gay and debonair, pulling Gustav Hamel, the irresistible boy, on to a Carlisle balcony to acknowledge the plaudits of the crowd—how one could multiply such instances!

In a curious undefined way the immense activities of those days are brought back, and immense they were from the fact that no one of the pioneers thought of, spoke of, or lived for anything else than flying. Aviation was their mistress, their religion, the very breath of their existence.

An indifferent world, which dismissed them as mad, or impious, or unscientific, according to its lights, enveloped them, but to the world they were more indifferent still. Conscious of the truth, full of sacred fire and almost insolently proud of their high calling, they wrought in obscurity and often in poverty—the most astonishing band of light-hearted fanatics who ever conquered an element.

All this and much more Major Turner has put into his fascinating book and, thanks to him, we can put it into the hands of the younger generation and say, "These were the men we knew."

H.

Internat Studiengesellschaft zur Erforschung der Arktis mit dem Luftschiff

Verh. der Versammlung in Berlin, November, 1926, edited by L. Breitfuss.

This Society was mooted two years ago with the object of furthering research in the Arctic regions by means of airships, and for stimulating scientific interest in the possibilities of polar exploration from the air. The present publication is a report of the papers read at the first general meeting of the Society, held in Berlin last year. It contains a foreword by Dr. Nansen. Most of the papers

are on geophysical subjects (including one in English by Sir Napier Shaw on the meteorology of the northern hemisphere), but two papers (by Captain Brüns and Dr. Bleistein respectively) deal with the design and construction of airships suitable for polar exploration. The book contains a number of useful maps and diagrams.

Untersuchungen über die Geschuindigkictsverteilung in Turbulenten Strömungen

By J. Nikuradse.

The author of this paper has investigated the turbulent flŏw at the end of pipes of a variety of cross-section and constructed velocity contours across the mouth of the pipe. Pitot tubes were used for the measurements, which were made with a view to testing the law $v = ay^{\frac{1}{7}}$ for the variation of velocity (v) with distance (y) from the surface. The results confirm this law, although measurements close to the bounding surface were not possible. For further details the original paper should be consulted.

THE JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY

With which is incorporated the Institution of Aeronautical Engineers

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Materials

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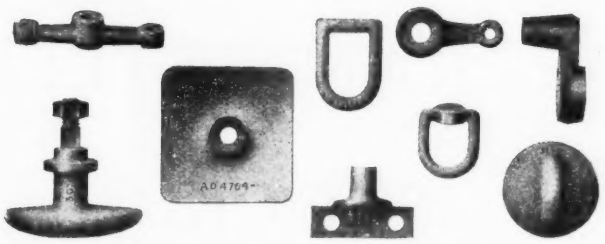

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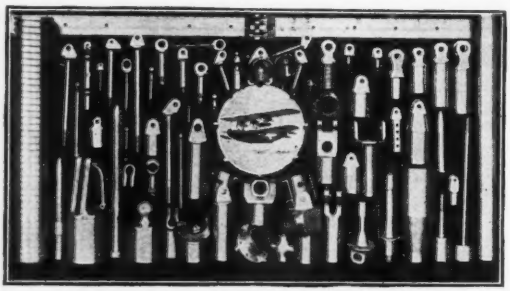

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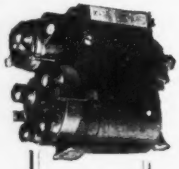

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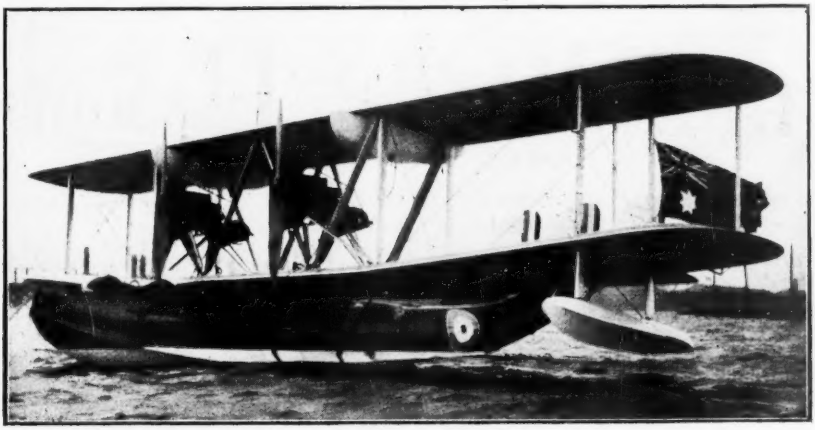

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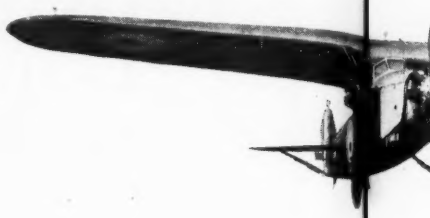

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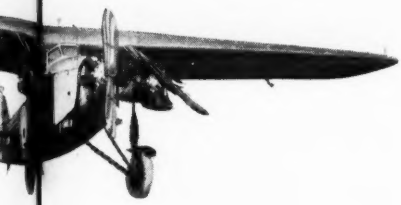

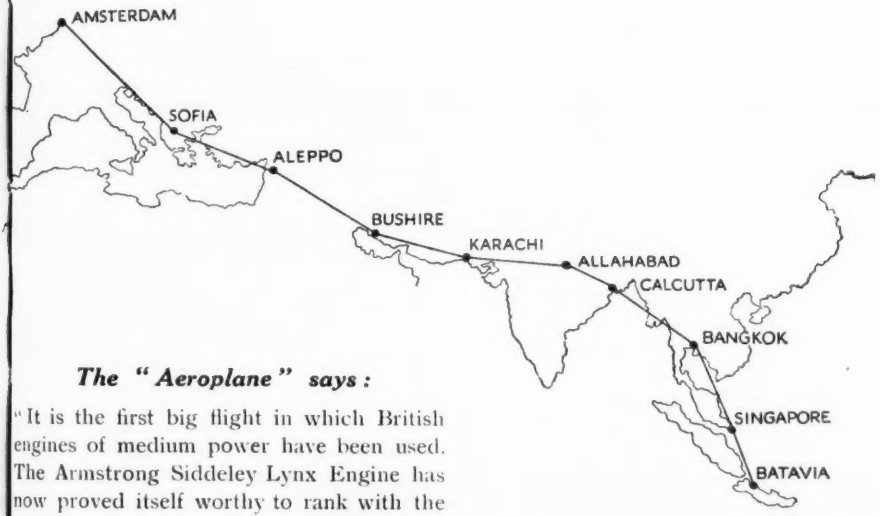

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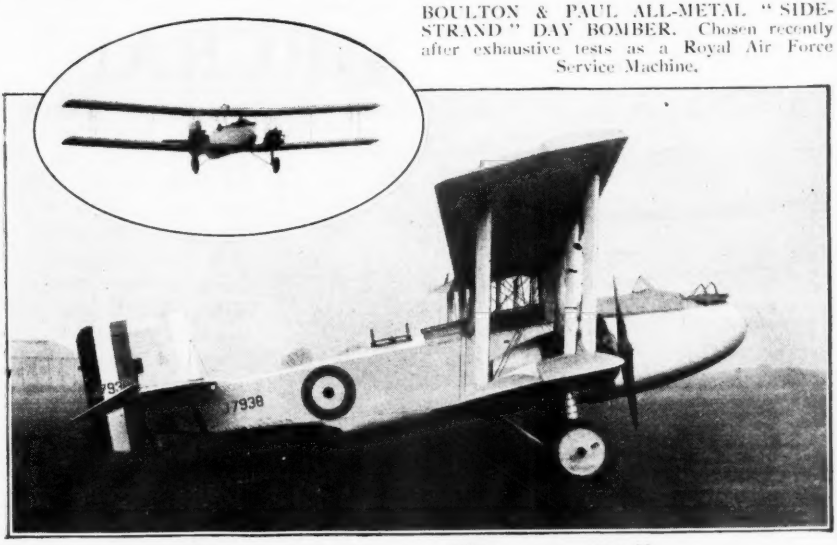

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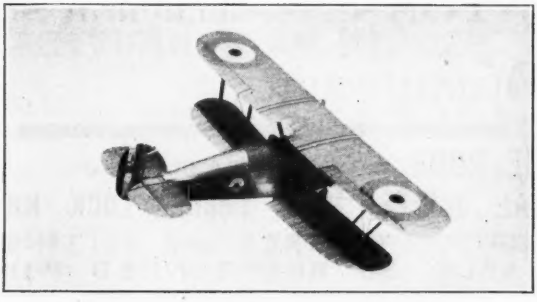

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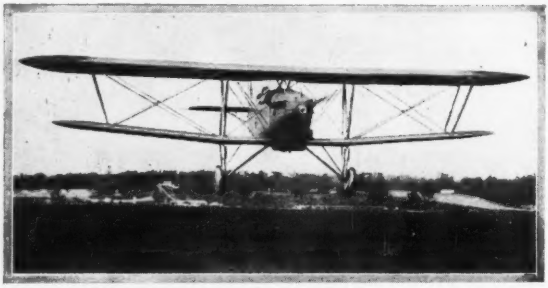

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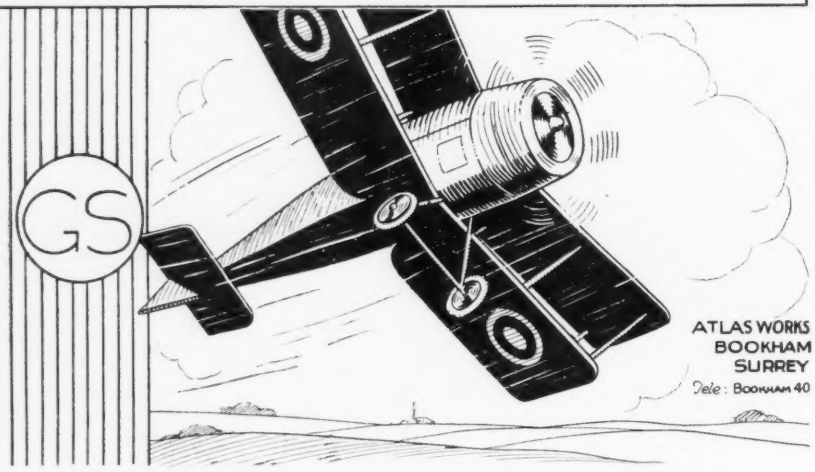

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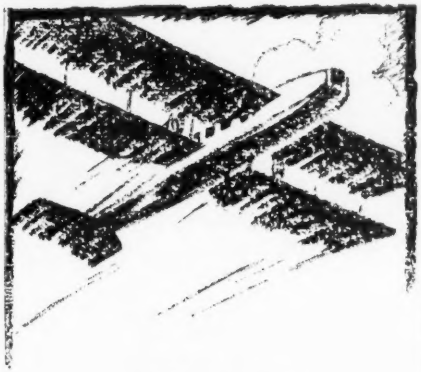

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